# HW5

Graph Theory (Fall 2019)
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Due: Tuesday, December 10, 2019, in class

1. There are 4 men a,b,c,d and 4 women A,B,C,D, and they have strict preference for each other for marriage, specified as follows.

   - a: $A > B > C > D$
   - b: $B > A > D > C$
   - c: $C > D > B > A$
   - d: $A > C > D > B$
   - A: $a > b > c > d$
   - B: $b > c > d > a$
   - C: $c > d > a > b$
   - D: $d > a > b > c$

   Run the Gale-Shapley stable marriage algorithm on this instance twice: once with the men proposing and once with the women proposing.

   Give an example of an unstable marriage for this setup. Why is it unstable?

2. Give an example of men and women with preferences for each other where there are two distinct stable marriages.

3. Give an example of a directed graph with capacities, and two vertices s, t, where there are 2 distinct flows which are both maximum.

4. Give an example of a directed graph with capacities, and two vertices s, t, where there are 2 distinct cuts which are both minimum.

5. Consider the directed graph with capacities on vertex set $\{1, 2, 3, 4, 5, 6, 7\}$, where for each $i < j$, there is an edge from $i$ to $j$ with capacity $j - i$.

   What is the maximum flow from vertex 1 to vertex 7 in this graph?

   Give a cut whose cut value equals the value of the flow you found above.

6. Consider the bipartite graph $G$ in the first part of problem 1 of HW4. Give a directed graph with capacities for which the max flow value equals the size of the maximum matching in $G$. Run the max flow algorithm to find a flow with this maximum value.

7. Use the determinant algorithm to check if the graph above has a perfect matching.

8. Let $G$ be a graph with two vertices $i, j$ and let $A_G$ be its adjacency matrix. Give a formula to compute the number of walks of length 2 between $i$ and $j$ in terms of $A_G$. Give a formula to compute the number of paths of length 2 between $i$ and $j$ in terms of $A_G$.