1.

Men proposing.

Round 1.

a, d propose to A, A chooses a.

b proposes to B, B accepts

c - - - - C, C accepts

So current pairings are

a – A
b – B
c – C

Round 2

d proposes to C, C rejects

Round 3

d proposes to D, D ~~rejects~~ accepts

Final Stable Marriage
a – A
b – B
c – C
d – D.

Women proposing can be done similarly.

Example of unstable marriage.

a - D

b - B

c - C

d - A.

Then a and A prefer each other to their current partners.

2.

a: $A > B > C$

b: $B > A > C$

c: $C > A > B$

A: $c > a > b$

B: $b > a > c$

C: $a > b > c$

then

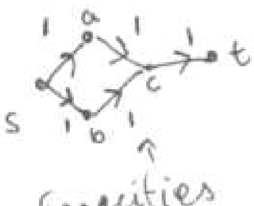
a - A
b - B
c - C

and

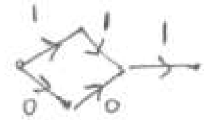
a - C
b - B
c - A

are both stable marriages.

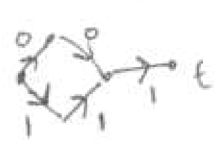
3.



Max flow 1



Max flow 2



Flow value = 1

This is max because cut value ({s,a,b,c}) = 1

4.

capacities.

$$s \xrightarrow{1} a \xrightarrow{100} b \xrightarrow{1} t$$

cut-value $(\{s\}) = 1$.

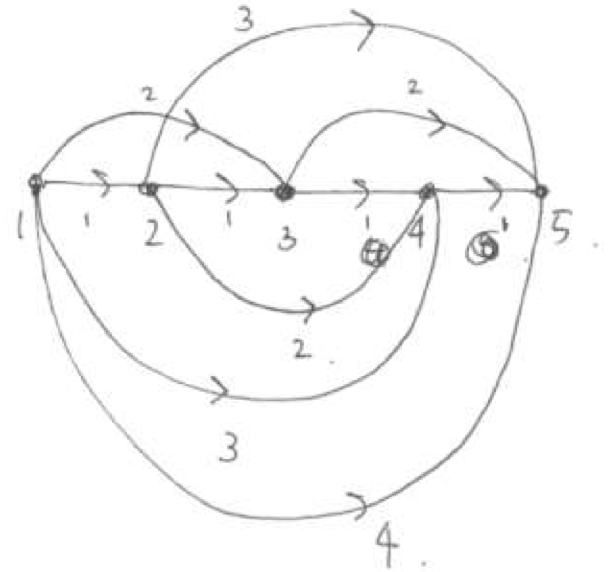
cut value $(\{s, a, b\}) = 1$.

These are min-cuts since there is a flow

$$s \xrightarrow{1} \circ \xrightarrow{1} \circ \xrightarrow{1} t$$

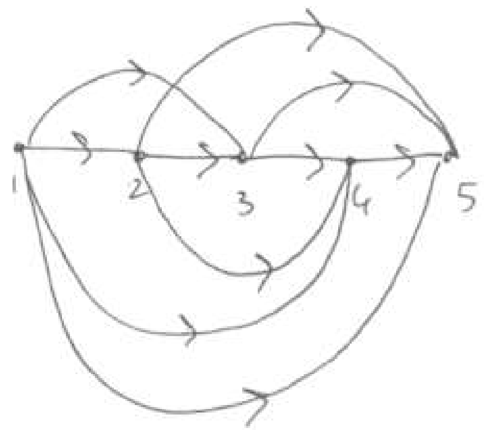
with value 1.

5.



Initial flow = all 0.

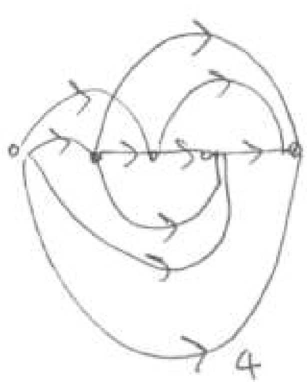
Initial Res. capacity. Graph.



Take path 1-5.

Send 4 units of flow on that edge.

New flow graph.



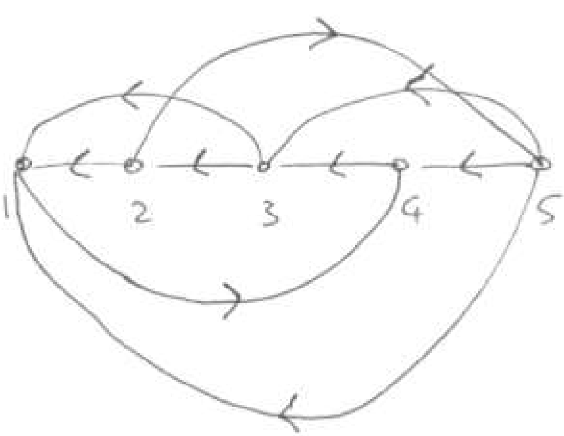
0 on all other edges.

New RCG : Same as before with 1-5 edge reversed now.

Then take path 1-3-5. Send 2 units of flow.

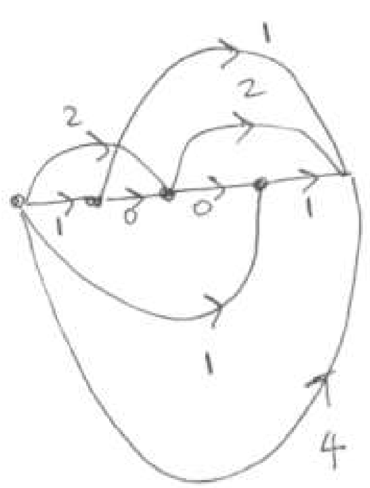
Then take path 1-2-3-4-5. Send 1 unit of flow.

At this point, RCG looks like.

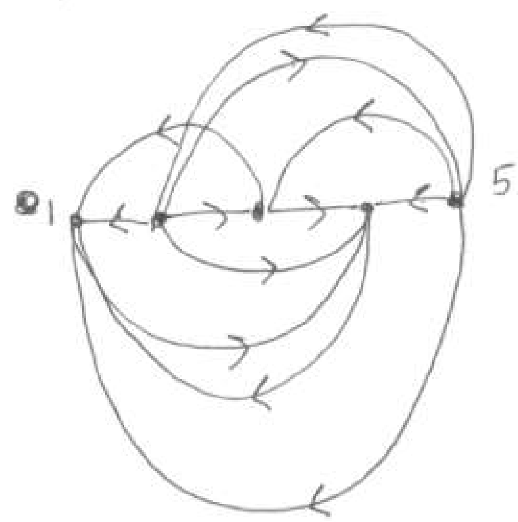


Take path 1-4-3-2-5. Can send 1 unit of flow.

New flow looks like.



New RCG looks like



No path from 1 to 5 in this RCG.

So we are done.

Max Flow value = 8.

Min Cut =

A = { vertices reachable from 1 } = {1, 4}.

Cut value ({1,4}) = 8.

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6 and 7 we did in class

8.

We know:

$$(A_G^2)_{ij} = \text{\# of walks of length 2 from } i \text{ to } j.$$

Which of these walks of length 2 are not paths?

Only walks $i\,w\,j$ where $j = i$.

# of these equals $(A_G^2)_{ii}$

so # paths of length 2 between i and j = $\begin{cases} 0 & \text{if } i = j \\ (A_G^2)_{ij} & \text{if } i \neq j \end{cases}$