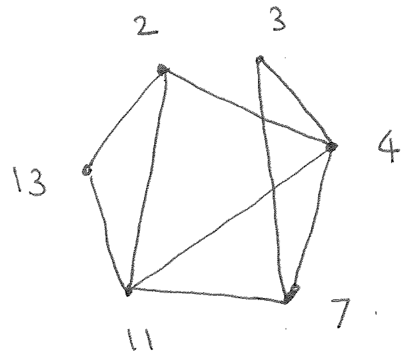
1.3



1.12

(a) $x\ u\ v\ r\ u\ v\ y$

(b) $v\ s\ t\ v\ w$

(c) No such path: neighbors of $r = \{u, s\}$ and z is not a neighbor of u or s.

(d) No such path.

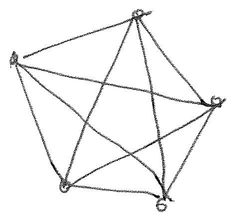
(e) $x\ u\ v\ t$ $\quad (d(x,t) = 3)$

(f) $x\ u\ r\ v\ y\ x\ u\ r\ v\ y\ x$

(g) $x\ u\ r\ s\ t\ w\ z\ y\ x$

(h) $r\ v\ y\ z$, $\operatorname{diam}(G) = 3$

1.15.



All distances have to be 1, because if some distance $d(x,y) \geq 2$, then the oddness condition means that $d(x,y) \geq 3$ and $d(x,y)$ is odd.

But if $x\ a_1\ a_2 \cdots a_\ell\ y$ is a path of length $d(x,y)$, then

$d(a_1, y) = d(x,y) - 1$ and is even and ≥ 2 (and hence $d(a_1, y) \neq 0$).

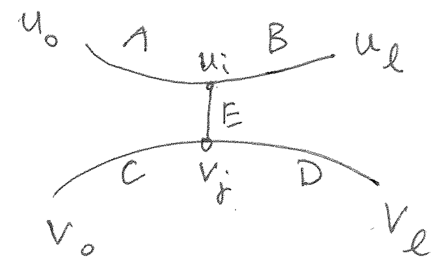
This is a contradiction.

So the graph has to be K_5.

1.17. (a). Suppose $u_0, u_1, \ldots, u_\ell$ and $v_0, v_1, \ldots, v_\ell$ are longest paths in G with no common vertices.

Let u_i, v_j be a pair of vertices that minimizes $d(u_i, v_j)$ $i \in \{0, 1, \ldots, \ell\}$ $j \in \{0, 1, \ldots, \ell\}$.

Then $d(u_i, v_j) > 0$ by assumption.



Let A, B, C, D, E be the paths shown.

(so A is the path $u_0, u_1, \ldots u_i$

B is $u_i\, u_{i+1} \cdots u_\ell$

and similarly for C, D.

E is the shortest path from u_i to v_j)

Note that $\text{length}(E) = d(u_i, v_j) > 0$.

Consider the path

(longer one of A, B)

followed by

E

followed by

(longer one of C, D).

The total length of this path is

$(\geq \ell/2) + (\geq 1) + (\geq \ell/2)$ which is

$\geq \ell + 1$, contradicting the hypothesis.

Let N_i = # vertices with degree i

2.3. 12 vertices

31 edges.

Handshake thm:

$$4 \cdot N_4 + 6 \cdot N_6 = 2 \cdot 31$$

$$N_4 + N_6 = 12$$

Solve:

$$N_4 = 5, \quad N_6 = 7$$

2.5. 25 vertices.

62 edges.

$N_4 = 2$

$N_6 = 11$

N_3, N_5 ?

$$N_3 + N_4 + N_5 + N_6 = 25$$

$$3N_3 + 4N_4 + 5N_5 + 6N_6 = 2 \cdot 62$$

Solve ..

2.7. Each edge has one endpoint in U and one endpoint in V.

So $\deg u$ counts the number of edges incident on u. $\sum_{u \in U} (\deg u)$ = total # of edges = m.

Similarly for V.