Homework 3

Design and Analysis of Data Structures and Algorithms (Spring 2012)
Rutgers University
Swastik Kopparty

Due Date: Monday, April 16, 2012

Questions

1. Read the article "College Admissions and the Stability of Marriage" by Gale and Shapley (available online at http://www.jstor.org/stable/2312726 - you can click "View PDF" to download it; you need to be on campus or using the VPN to access the article).

   "Amuse yourself" by solving the example instance given on page 13.

2. Design a finite automaton which can be used to find all occurrences of the string "abcaba" in a given input string.

   Write down the sequence of states that this automaton visits when given as input "abacaabaababcababcaba".

3. Given two sequences $a_0, \ldots, a_n$ and $b_0, \ldots, b_m$ (with $m < n$) we want to compute, for each $j \in \{0, 1, \ldots, n-m\}$, the quantity:

$$c_j = \sum_{i=0}^{m} a_{j+i} b_i.$$

   (This showed up as a subroutine when we did string matching with don't-cares).

   Show how one can use the algorithm for multiplying polynomials of degree n in $O(n \log n)$ time to compute all the c_j in time $O(n \log n)$.

   Bonus: by breaking the sequence $a_0, \ldots, a_n$ into blocks of suitable size, show how to compute all the c_j in time $O(n \log m)$.

4. We have m servers and n clients. Each client gets served by a uniformly random server.

   - Suppose $n = o(m \log m)$. Show that with probability $1 - o(1)$, there exists some server who does not serve any clients.
   - Suppose $n = \omega(m \log m)$. Show that with probability $1 - o(1)$, every server serves at least 1 client.

5. There are n people $\{1, 2, \ldots, n\}$, and each person is either a Democrat or a Republican. Suppose 1/2 of the people are Democrats. We consider three different ways of "polling" these people.

   - Pick t people independently and uniformly at random from amongst all the people. Using the Chernoff bound, show that the probability that more than 3/4 of the people we pick are Democrats is at most $2^{-\Omega(t)}$.

- Now we pick a random $i \in \{1, 2, \ldots, n\}$, and then pick the people $\{i, i+1, \ldots, i+t-1\}$ (all these numbers taken mod n). Using the Markov bound, show that the probability that more than $3/4$ of the people we pick are Democrats is at most $2/3$.

  Show that there are situations where this probability actually could be as large as $2/3 - O(1/n)$.

- Assume n is prime. Now we pick independent random $i, j \in \{1, 2, \ldots, n\}$, and then pick the people $\{i, i+j, i+2j, \ldots, i+(t-1)j\}$ (all these numbers taken mod n). Recall that these numbers have a pairwise independent distribution.

  Using the Chebhyshev bound, show that the probability that more that $3/4$ of the people we pick are Democrats is at most $O(\frac{1}{t})$.

6. **Midterm Makeup:** For any question of the midterm, if you want to redo your solution to that question, you can get up to $1/2$ the remaining points back. Submit any redone questions along with HW3.

   Specifically, if the question is worth a points, and in the midterm you got b points for it, and your redone solution gets c points, then the number of points that you will get finally for that question is $\max\{b, (b+c)/2\}$.