

Homework 1

Theory of Numbers (Fall 2014)
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Due Date: Thursday, September 18, 2014

Questions

1. Guess, and then prove, a formula for:

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{(n-1) \cdot n}.$$

2. Show that every integer can be written as a sum of distinct Fibonacci numbers.

(For example: 75 can be written as a sum of distinct Fibonacci numbers, since $75 = 55 + 13 + 5 + 2$. But the representation of 75 as $55 + 8 + 8 + 3 + 1$ does not count, since the summands are not distinct.)

3. Define a sequence a_n as follows:

- $a_0 = 1$.
- For each $n \geq 0$, define $a_{n+1} = a_n + \frac{1}{a_n}$.

Prove by induction that $a_n \geq \sqrt{2n}$.

Bonus: Prove by induction that $a_n \leq \sqrt{3(n+1)}$.

4. Convert 228 in base 9 to base 7. Show all your work!