This is homework 2 for Math 114s in winter quarter of 2016. It is due Friday January 22nd in class.

Do the following exercises from the book: x4.5, x4.10-16, x4.25-27.

(1) Let A and B be sets. If A and B are disjoint, then there are obvious injections $f : A \to A \cup B$ and $g : B \to A \cup B$ such that $f``A$ and $g``B$ are disjoint and $f``A \cup g``B = A \cup B$.

Now there is a problem if A and B are not disjoint. Find a set X and injections $f : A \to X$ and $g : B \to X$ such that $f``A$ and $g``B$ are disjoint and $f``A \cup g``B = X$. This set X is sometimes written $A \sqcup B$ and is called the disjoint union of A and B.

(2) Generalize the previous exercise to arbitrary unions $\bigcup_{i \in I} A_i$. The set is sometimes written $\bigsqcup_{i \in I} A_i$ and is called the disjoint union of the A_i.

The sets from this exercise allow us to take the union in a way where each element in the union remembers which set it came from. We will make use of this in the next exercise.

(3) Let $(L_0, <_0), (L_1, <_1), \ldots (L_n, <_n)$ be orderings. Define $(L_0, <_0) + (L_1, <_1) + \cdots + (L_n, <_n)$ to be the ordering with underlying set $L_0 \sqcup L_1 \sqcup \cdots \sqcup L_n$ ordered by $l < l'$ if and only if there is an $i \leq n$ such that $l, l' \in L_i$ and $l <_i l'$ or $l \in L_i$ and $l' \in L_j$ where $i < j$.

Suppose that $(W_0, <_0), (W_1, <_1), \ldots (W_n, <_n)$ are wellorderings. Show that $(W_0, <_0) + (W_1, <_1) + \cdots + (W_n, <_n)$ is a wellordering.

(4) Suppose that $(L_0, <_0)$ and $(L_1, <_1)$ are orderings. Let $(L_0, <_0) \times (L_1, <_1)$ be the ordering with underlying set $L_0 \times L_1$ where $(l_0, l_1) < (l_0', l_1')$ if and only if either $l_1 = l_1'$ and $l_0 <_0 l_0'$ or $l_1 <_1 l_1'$.

Suppose that $(W_0, <_0)$ and $(W_1, <_1)$ are wellorderings. Show that $(W_0, <_0) \times (W_1, <_1)$ is a wellordering.

(5) Prove that every infinite ordinal can be written uniquely as the sum of a limit ordinal and a finite ordinal.