

**FORCING HINTS**  
**DAY 7**

**Hint 1.** For the (first) Lowenheim-Skolem theorem, Itay suggested that you prove the following:

Let  $X \subseteq U$  be infinite sets, and let  $f: U^{<\omega} \rightarrow U$  be a function. ( $U^{<\omega}$  is the set of finite subsets of  $U$ .) Prove that there is  $Y \subseteq U$  so that  $X \subseteq Y$ ,  $|Y| = |X| \cdot \aleph_0$ , and  $Y$  is closed under  $f$ , meaning that  $f''Y^{<\omega} \subseteq Y$ .

The ideas from proving this fact appear in the proof of the Lowenheim-Skolem theorem.