**Hint.** For problem 1 it would be helpful to show that the union of  $\kappa$ -many sets of size  $\kappa$  has size  $\kappa$ .

**Hint.** For problem 2, you need to do a similar diagonalization to those that we did in class. At stage n you must have made promises which deal with the first n sets.

**Hint.** For problem 3, the diagonalization is similar. To show that t is regular, suppose that it is singular and then break the tower in to smaller chunks.

**Hint.** For problem 4, it is enough to make an almost disjoint family of some countable set. Such a family can easily be copied to one on  $\omega$ .