Hint. For problem 1 it would be helpful to show that the union of $\kappa$-many sets of size $\kappa$ has size $\kappa$.

Hint. For problem 2, you need to do a similar diagonalization to those that we did in class. At stage $n$ you must have made promises which deal with the first $n$ sets.

Hint. For problem 3, the diagonalization is similar. To show that $\mathfrak{t}$ is regular, suppose that it is singular and then break the tower in to smaller chunks.
Hint. For problem 4, it is enough to make an almost disjoint family of some countable set. Such a family can easily be copied to one on $\omega$.

