FORCING EXERCISES
DAY 9

Problem 1. *Show that if $\mathbb{P}$ is a splitting partial order and G is M-generic over $\mathbb{P}$, then $G \notin M$. Hint: Suppose not. Show that $\mathbb{P} \setminus G$ is dense.*

Problem 2. *Let $\mathbb{P} \in M$ be a splitting partial order. Show that there are continuum many M-generic filters over $\mathbb{P}$.*

Problem 3. *Let M be a countable transitive model of ZFC, and let $\mathbb{P}$ be a poset with $\mathbb{P} \in M$. Suppose that G is a subset of $\mathbb{P}$ that intersects every dense set belonging to M, that whenever $p \in G$ and $p \leq q$ then $q \in G$, and that any two elements of G are compatible. Then show that G is a filter. (Another way of phrasing this problem is to say that if we weaken the condition $(\forall p \in G)(\forall q \in G)(\exists r \in G) r \leq p, q$ to $(\forall p \in G)(\forall q \in G)(\exists r) r \leq p, q$ for generic filters we still get the same notion).*

Problem 4. *Assume that $\mathbb{P} \in M$ and $\mathbb{P}$ is infinite. Show that there is an $H \subseteq \mathbb{P}$ so that $M[H]$ is not a model of ZFC. (Hint: Recall that $M[H]$ and M have the same ordinals. We never used the fact that H is a generic filter to prove that!).*

Problem 5. *Let $\sigma, \tau \in M^{\mathbb{P}}$. Construct a name $\pi \in M^{\mathbb{P}}$ so that regardless of G, $i_G(\pi) = i_G(\sigma) \cup i_G(\tau)$. (Hint: this can be set up to work for any subset G of $\mathbb{P}$).*

Problem 6. *Let $\sigma \in M^{\mathbb{P}}$. Construct a name $\pi \in M^{\mathbb{P}}$ so that regardless of G, $i_G(\pi) = \bigcup(i_G(\sigma))$. (Hint: this can be set up to work for any filter; you don't have to use the fact that G is generic).*

Problem 7. *Let $\tau \in M^{\mathbb{P}}$ with $dom(\tau) \subseteq \{\check{n} : n \in \omega\}$. Construct a name $\sigma \in \mathbb{P}$ so that regardless of G, $i_G(\sigma) = \omega \setminus i_G(\tau)$. (Hint: you will have to make use of the genericity of G).*