FORCING EXERCISES
DAY 4

The exercises are not ordered by level of difficulty.

Problem 1. *Let* $\mathbb{P}$ *be as in the proof that* MA *implies* $\mathfrak{b} = 2^{\aleph_0}$.

(1) *Give an explicit example of maximal antichain in* $\mathbb{P}$.
(2) *For each* $k \in \omega$, *show that* $\{(p, A) \mid k \in \operatorname{ran}(p)\}$ *is not dense.*

Problem 2. *Prove the Theorem 4.3 from the notes. (There are hints in the notes.)*

Problem 3. *Show that* MA *implies* $\mathfrak{s} = 2^{\omega}$.

Problem 4. *Show that* MA *implies* $\mathfrak{t} = 2^{\omega}$.

Problem 5. *Suppose that* $\{A_n \mid n < \omega\} \subseteq \mathcal{L}$ *and for all* n, $A_n \subseteq A_{n+1}$. *Show that* $\mu(\bigcup_{n<\omega} A_n = \lim_{n<\omega} \mu(A_n)$.

Problem 6. *We define a poset* $\mathbb{P}$. *Conditions are subsets of the interval* $(0, 1)$ *which have positive Lebesgue measure and they are ordered by* $p \leq q$ *if and only if* $p \subseteq q$.

(1) *Give a characterization of* $p \perp q$ *for* $p, q \in \mathbb{P}$.
(2) *Show that if* $\{p_n \mid n < \omega\}$ *is an antichain, then* $\mu(\bigcup_{n<\omega} p_n) = \sum_{n<\omega} \mu(p_n)$.
(3) *Show that* $\mathbb{P}$ *is ccc.*

Problem 7. *Do the following.*

(1) *Show that every open set can be written as the union of open intervals with rational endpoints.*
(2) *Show that there are exactly* $2^{\aleph_0}$ *open sets.*

Problem 8. *Show that there are exactly* $2^{\aleph_0}$ *many Borel sets.*

Problem 9. *Show that there are Lebesgue measure zero sets which are not Borel. (Hint: Think about the Cantor set.)*