Problem 1. *Let κ be an infinite cardinal. Show that κ^+ is a regular cardinal.*

Problem 2. *A family $\mathcal{F} \subseteq \mathcal{P}(\omega)$ is called a splitting family if for every infinite $X \subseteq \omega$ there is a set $A \in \mathcal{F}$ such that $|X \cap A| = |X \setminus A| = \omega$. The splitting number $\mathfrak{s}$ is the minimal cardinality of a splitting family. Show that $\omega < \mathfrak{s} \leq 2^\omega$.*

Problem 3. *Let A, B be subsets of ω. We write $A \subseteq^* B$ when $A \setminus B$ is finite. A sequence $\langle A_\alpha \mid \alpha < \kappa \rangle$ of distinct infinite subsets of ω is a tower if $A_\beta \subseteq^* A_\alpha$ whenever $\alpha < \beta$. The tower number $\mathfrak{t}$ is the minimal length of a maximal tower (a maximal tower is one for which no further set is almost contained in every member of the tower).*

(1) *Show that $\omega < \mathfrak{t} \leq 2^\omega$.*
(2) *Show that $\mathfrak{t}$ is a regular cardinal.*

Problem 4. *Show that $\mathfrak{b}$ is a regular cardinal.*

Problem 5 (*). *Show that there is a MAD family of cardinality $2^{\aleph_0}$.*