The Spirit of Calculus

Quest University Canada Block 2, Fall 2016

> *The calculus was the first achievement of modern mathematics and it is difficult to overestimate its importance. I think it defines more unequivocally than anything else the inception of modern mathematics* - John von Neumann

In the 17th century, two mathematicians – German Gottfried Leibniz and Englishman Isaac Newton – simultaneously discovered an intimate connection between two seemingly unrelated problems: measuring changing quantities and finding areas of curved shapes. This discovery formed the basis of calculus, a subject which stands as one of the most important fields of mathematics today.

Calculus has earned its reputation because it provides us with tools that can be applied to solve problems in every branch of science that would be impossible to answer without it. For example, calculus allows us to easily find the greatest possible profit or land size under given conditions, to make solid predictions based on probabilities, and to compute geometric quantities like area, volume, and surface area with ease.

The applicability of calculus was recognized long before its theory was sorted out. In fact, the foundations of Leibniz's and Newton's calculus were quite shaky: both mathematicians based their work on the idea of infinitely small quantities but could not provide a definition of them. This opened the subject up to attack from critics such as Bishop Berkeley, who argued that accepting calculus required more faith than theology. It took over two hundred years for mathematicians to come to grips with the idea of the infinite and for the subject to be placed on a firm foundation.

In this class, we will focus on *differential calculus*, the branch of calculus that is motivated by the problem of measuring how quantities change. We will study both the theory of calculus and its applications, in light of the long journey to its modern form. The primary Learning Goals for this course are as follows.

1. Understand multiple representations and important properties of functions, derivatives, and integrals, and the relationships between them
2. Communicate, both verbally and in writing, key ideas from calculus with clarity, precision, and confidence
3. Apply knowledge of calculus to solve a variety of complex problems
4. Improve your ability to read and learn from mathematical texts

Course Information

Instructor:	Dr. Sarah Mayes-Tang	Dates:	Block 2, Fall 2016
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Course Topics

Calculus is divided into two branches: differential and integral calculus. The goal of this course is to explore the fundamental ideas and tools of differential calculus and work our way towards the Fundamental Theorem of Calculus, which unites the two branches of the subject together.

We will work to answer the following questions.

- How do we use mathematics to describe related quantities?
- What is an instantaneous rate of change?
- How do we work with the infinitely small?
- How can we use mathematics to describe change?
- How are rates of change computed?
- What problems does the derivative help us solve?
- What is the relationship between rates of change and areas of curved shapes?
- What is Calculus?

Textbook and Software

We will be using the text *Single Variable Calculus: Early Transcendentals* by James Stewart. You may use either the 7th edition or the 8th edition. The 8th edition is available at the Quest University bookstore (the 7th edition has been discontinued, but you may find some used copies on campus).

We will also be making occasional use of the open-source software *Geogebra*, available at `www.geogebra.org`. Since it may also help you with your homework, I recommend that you download it at the beginning of the course.

Course Requirements

Learning calculus requires a lot of hard work. To demonstrate that you have met the Learning Goals and pass this course, you will be required to:

- Complete (many!) problems and read mathematics in the Daily Homework
- Present problems in class and participate in discussions
- Perform well on 3 exams, and demonstrate understanding on exam revisions
- Write clear and comprehensive solutions to problems on Synthesis Assignments

Daily Homework

> *You have to be confused before you can reach a new level of understanding anything.* – Dudly Herschbach

The Daily Homework will consist of readings and several problems. The problems will be of varying difficulty: some will be easy warm-ups to the reading or simple computations, while others will involve complex situations and problem solving skills. I expect that you will spend nearly all of your study time on these daily assignments. We will spend significant amount of class time reviewing and discussing these problems.

Your notes on the assigned problems should contain enough detail to be easily understood by you at a later time. For example, you will want to refer to these problems when preparing for exams, and your notes will help you to swiftly present and discuss solutions during class. While you do not need to write your solutions in complete sentences, they should be easy to follow and clear. If you decide to add to these notes during class before they are collected, you must use a separate colour to clearly distinguish what you prepared before class.

I will collect, grade, and return these assignments each day. You will receive one point (a ✓) for a question when it appears that you have put adequate thought and work into it (not necessarily if it is right or wrong). On most days I will just glance at your solutions, but I will occasionally read them in detail to ensure that you are actually working through the problem.

Reading

The ability to understand mathematical texts is an important skill for *any* future mathematical study. This skill is vital for at least three reasons.

1. **Future Learning**. When you need to learn a mathematical concept on your own, your main resources will be written.

2. **Efficiency**. In an ideal world we might try to discover mathematics itself, but this would be impractical. Calculus, for example, took hundreds of years to develop and another 200 years to gain a firm footing. The great abundance of mathematical writing available allows us to learn from the experts.

3. **Learning to Communicate**. Just as reading many stories makes you a better storyteller, reading a lot of mathematics makes you a better mathematical communicator.

During Week 1 we will be discussing guidelines and tips for reading mathematics effectively. To provide motivation to keep up with the reading, I may randomly choose one student to summarize each reading assignment during class.

Presenting Solutions[1]

> *The intelligence is proved not by ease of learning, but by understanding what we learn.* – Joseph Whitney

[1] from Carol Schumacher

We have a lot of work to accomplish this block and your contributions during class are crucial to the success of others. Student presentations of homework problems will be a centrepiece of the class, so you should take them seriously. Here are some things to know about making presentations in this class.

- The purpose of class presentations is not to prove to the instructor that you have done the problem. It is to make the ideas of the proof clear to the other students.
- You must use proper English and mathematical grammar during presentations
- Presenters should explain their reasoning as they go along, not simply write everything down and then turn to explain.
- Fellow students are allowed to ask questions at any point and it is the responsibility of the person making the presentation to answer those questions to the best of his or her ability.
- Presentations are directed at students, so the presenter should look at students to be able to see how well they are following the presentation.

I will use an online poll to organize volunteers for presenting problems. For each problem, you will be asked to indicate whether you are happy to present the problem ('Yes'), if you are willing to present the problem but would rather not ('(Yes)'), or if you do not want to present the problem ('No'). The more problems that you are willing to present, the more likely you are to get your preferences.

The following scale will be used to grade your presentations. If you'd like to feedback how your presentations are going or how you can improve, please ask me the day of or shortly after your presentation.

Mark	**Criteria**
4	Completely correct and clear solution
3	Solution has minor technical flaws, unclear language, or lacking details. Essentially correct.
2	A partial explanation is provided but a significant gap still exists
1	Minimal progress has been made; includes relevant information that could lead to a solution
0	Completely unprepared

Synthesis Assignments

> *If I have made any valuable discoveries, it has been owing more to patient attention than to any other talents* - Sir Isaac Newton

Each day, you will be assigned one Challenge Problem to solve and submit for detailed feedback at the end of the week; the collection of Challenge Problems from each week will form the Synthesis Assignment. As indicated by their title, these questions are intended to be challenging: they will not follow from templates in a textbook and cannot be solved by a computer. The problems are designed to develop your problem-solving abilities and reinforce your understanding of the content of the course. As in real-life problems, you will not always be given exactly the same information as you need to solve a them.

One of the best ways to improve your writing for mathematics, science, and beyond is to practice writing mathematics. Therefore, the synthesis assignments are designed to give you an opportunity to practice and improve your mathematical writing. Writing clear mathematics is not only important in math classes: it can help to improve your mathematical thinking and your ability to communicate ideas from other fields clearly. I have high expectations for your writing; during the first week of the block we will be discussing how to prepare solutions for these assignments. Please pay careful attention to these guidelines before submitting your first assignment. You will have the opportunity to receive feedback on a written solution before your first assignment is due.

You may discuss these problems with classmates but you must write your own solutions independently, without copying. You may choose to either write or type your solutions *neatly*. Unless you are a proficient user of LaTeX or cannot write neatly by hand, it will be easier for you to handwrite your assignments.

All homework is due at the beginning of class, and all deadlines are absolutely firm. I will not accept late homework since we will discuss these problems in class and keeping up with deadlines will encourage you to keep up with course material.

Examinations and Revisions

> *Seeing much, suffering much, and studying much are the three pillars of learning.* - Benjamin Disraeli

Exams will help to accomplish the Learning Goals of this course in several ways:

- Encourage you to push yourself to understand difficult concepts and to complete *many* challenging problems
- Give you authentic feedback on your independent problem solving abilities
- Ensure that you have successfully passed major landmarks of conceptual understanding, and are prepared for further challenges

To help you learn from the two Mid-Block Exams, I will give you the opportunity to submit corrections after I have graded them. If your corrections are well-written and accurate, you will earn partial credit for that problem on the exam.

There will be three exams in total, held during Weeks 2, 3, and 4 of the block (see the Schedule for details).

Participation

> *There is a difference between not knowing and not knowing yet.* -Sheila Tobias

To learn calculus and to contribute to the learning of your peers, you must be active and engaged during class. During a presentation you are not off the hook just because someone else volunteered. Even if you are sitting down, you are responsible for contributing questions and comments to help clarify what is being presented.

It is important to note that you should be just as eager to offer questions about concepts that you understand as those that you do not understand. For example, if you have done the problem and understand it well, you can often make suggestions that help clarify the wording or structure of a presentation.

Here are some additional guidelines for participation.

- **Actively participate in course discussions,** asking questions, offering comments, and listening carefully to what others say. If you have a question about something, please ask! There are likely other people who have the same question.
- **Be respectful of other class members and maintain a collaborative environment.** Contributing to a class discussion does not mean talking a lot. You should listen carefully to others' ideas and be careful about offering a critique. When you do object to others' ideas, be kind.
- **Respect course policies.** I have set course policies that help the class to run smoothly for everyone. Please respect these rules.
- **Note Taking.** You should take notes in class that you can refer back to later. I recommend that you use a binder to organize your notes, as there will be frequent in-class handouts and worksheets that would be difficult to corral into a notebook.

Academic Integrity

While googling a Challenge Problem or trading solutions with a classmate may seem like good strategies for doing well in this class, these actions will prevent you from learning material, refining your problem-solving skills, and developing self-sufficiency and self-esteem.

The consequences for cheating are severe. *Any* blatant academic dishonesty will result in failure of the course and immediate reporting to the Chief Academic Officer.

The following actions are *not* considered cheating.

- Discussing questions from problem sets with classmates, building off of each others' ideas
- Using online resources to help you understand the content of the course or practice problems (e.g. problems that you do not submit)

The following actions *are* considered cheating.

- Looking for solutions to Synthesis Assignment or Exam problems online (e.g. by searching or posting on a message board).
- Copying the writing or explanations of mathematical work from someone else
- Speaking to classmates about exam solutions before submitting corrections

Looking for solutions to Daily Homework problems online or copying from classmates is a very bad way to learn calculus, but it is impossible for me to prevent this type of behaviour. Therefore, I ask that if you copy something, make sure to acknowledge it.

These examples are not comprehensive; if you have questions about whether something is considered cheating, please speak with me first.

Peer Tutor

Aaron Slobodin the subject specialist peer tutor for this course. He is a calculus expert who will hold review sessions through the block. The style of these review sessions will depend on your needs and preferences, but generally they will cover difficult concepts, practice problems, tricky questions on *completed* problem sets, and prerequisite skills. They will be (tentatively) scheduled at the following times.

- Monday and Wednesday, 7 pm (in the classroom)

I *strongly* encourage you to attend these review sessions.

Grading

> *While grades are (one) measure of progress, they are not a measure of promise.*
> –Francis Su

Your final grade will be calculated as follows.

20%	Daily Homework and Participation
20%	In-Class Presentations
20%	Weekly Synthesis Assignments
40%	Exams
	First Exam: 10%
	Second Exam: 10%
	Third Exam: 20%

The course grading scale is:

A	93-100%	B	83-86%	C	73-76%
A-	90-92%	B-	80-82%	C-	70-72%
B+	87-89%	C+	77-79%	D	60-69%

Narrative Evaluation

Any student at Quest can request a narrative evaluation (e.g. a written paragraph) in addition to their letter grade in any course. A narrative evaluation will give you more comprehensive feedback that you can learn from and additional information to present employers and graduate schools. If you wish to take advantage of this option, you have until the end of the 6th day of a course to sign up on the Registrar's Office Portal site.

Disability Accommodations

Students experiencing disabilities should schedule a meeting with me to discuss their Academic Accommodation Plan.

Additional Course Policies

- I expect that you will attend every class session. If you must miss a class for a valid reason (such as illness or a family emergency), please let me know *before* class. For

every class that you miss without a valid reason, 5% will be deducted from your *full* course grade.

- Please be on time to class. If you are late to more than 2 class sessions, I will deduct 1% from your course grade for each additional class that you are late to.
- Bring pencils, paper, textbook, and a scientific calculator to every class. I will let you know prior to class if you need to bring your laptop.
- If you do bring a laptop to class, keep it shut when not using it for mathematics.
- Do not use your cell phone during class. If you are on your cell phone, you will be asked to leave.
- Always be respectful in your speaking and actions. Do not use profanity.
- Office hours: The times of my office hours will be posted on the course website weekly. I expect that you will come to at least one office hour within the first two weeks of the course to help me get to know you and your goals better.
- If you are not able to make my drop-in office hours, or need to speak with my privately, please e-mail me to arrange an appointment at least 2 days before the desired meeting time. My schedule fills up quickly so while I will do my best to accommodate emergency requests, I may not be able to.
- E-mail: I respond to emails twice a day on weekdays (generally at the beginning and end of the day), and sporadically during weekends. Please do not e-mail me with questions that may be easily answered by looking at this syllabus, the course website, or asking other members of the class. I expect that you will be polite and use proper English grammar.
- Please do not bring food into the classroom. You may bring drinks.

Schedule

Day		Work Due	Topics & Activities	Do
Mon, Oct. 3	**1**		Introduction to calculus Investigating changing quantities Daily Homework Workshop [Ch. 1]	Daily Homework
Tues, Oct. 4	**2**	Daily Homework	Building mathematical models Rates of change Using rates of change to approximate Mathematical Writing Workshop [Ch. 1]	Daily Homework Prepare Formal Solution for Review
Wed, Oct. 5	**3**	Daily Homework Formal Solution	Instantaneous rates of change The derivative in 4 representations Idea of limits Reading Mathematics Workshop [§2.1, §2.7]	Daily Homework
Thurs, Oct. 6	**4**	Daily Homework	Limits graphically and numerically The limit laws Differentiability and continuity Exam Preparation Workshop [§2.2, §2.3a, §2.5]	Daily Homework Compile Synthesis Assignment

Fri, Oct. 7	**5**	Daily Homework Synthesis Assignment 1	Finding limits algebraically Comparing functions to find limits Dealing with the infinitely small Using the derivative to make predictions [§2.3b, §2.7]	Daily Homework Prepare for Exam 1
Monday, October 10: Thanksgiving (no class)				
Tues, Oct. 11	**6**	Daily Homework	The derivative as a function Derivatives graphically and algebraically [§2.8]	Daily Homework Prepare for Exam 1
Wed, Oct. 12	**7**	Exam 1 Daily Homework	Computing derivatives: polynomials, exponential functions, and products Using derivatives [§3.1, §3.2]	Daily Homework
Thurs, Oct. 13	**8**	Daily Homework	Computing derivatives: trig functions, quotients, and compositions [§3.2b, §3.3, §3.4]	Daily Homework Complete Exam Corrections Compile Synthesis Assignment

Fri, Oct. 14	**9**	Daily Homework Exam Corrections Synthesis Assignment 2	Computing derivatives: logarithmic functions Differentiating implicitly defined functions Solving problems involving related rates [§3.5, §3.6a, §3.9]	Daily Homework Study for Exam 2
Mon, Oct. 17	**10**	Daily Homework Synthesis Assignment 2	What is optimization? Locating extrema [§4.1]	Daily Homework Study for Exam 2
Tues, Oct. 18	**11**	Daily Homework Exam 2	Maximums, minimums, and derivative tests Finding global extrema Studying families of functions with calculus [§4.3, §4.5, §4.6]	Daily Homework
Wed, Oct. 19	**12**	Daily Homework	L'Hospital's pulley problem Applied optimization problems Limits involving ∞ [§4.7, §2.6a]	Daily Homework Complete Exam Corrections
Thurs, Oct. 20	**13**	Daily Homework Exam Corrections	Applied optimization problems, continued Finding distance from velocity [§4.7, §5.1a]	Daily Homework Compile Synthesis Assignment

Fri, Oct. 21	**14**	Daily Homework Synthesis Assignment 3	The area problem The definite integral [§5.1b, §5.2]	Daily Homework Prepare for Final Exam
Mon, Oct. 24	**15**	Daily Homework	Interpreting definite integrals Putting it together: The Fundamental Theorem of Calculus [§5.4b]	Daily Homework Prepare for Final Exam
Tues, Oct. 25	**16**	Daily Homework	What is Calculus? Review of the Course	Prepare for Final Exam
Wed, Oct. 26	**17**		Final Exam	