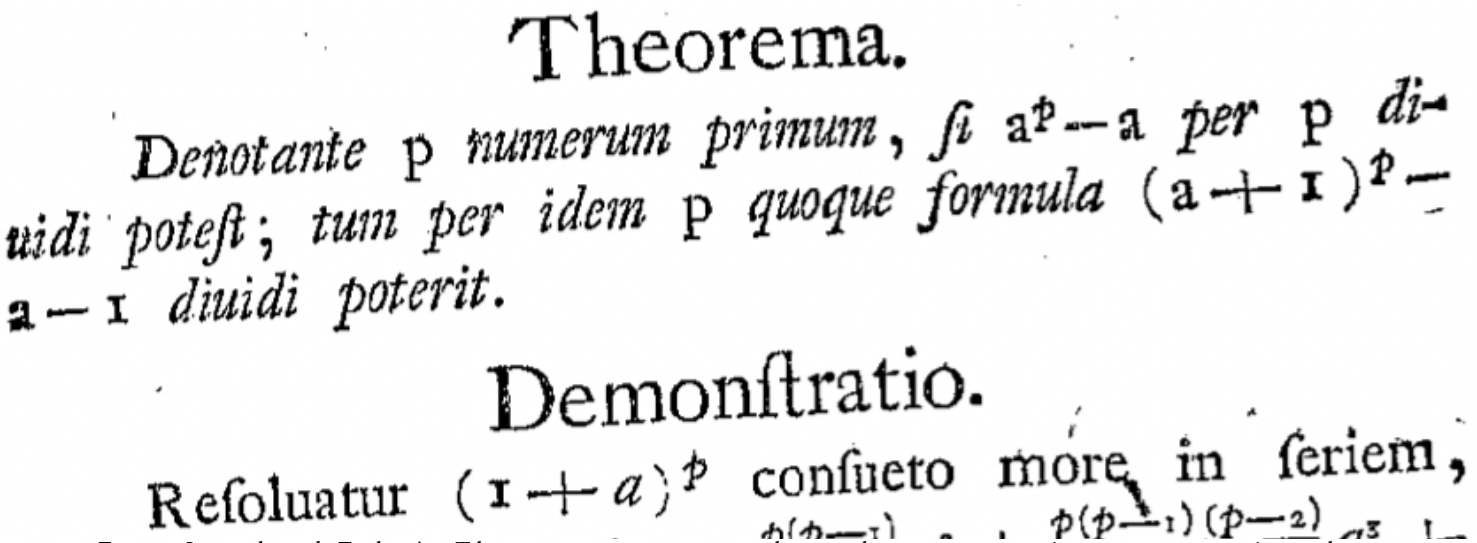
MAT246 LEC0101

how do we create mathematical masterpieces?



Theorema.

Denotante p numerum primum, ſi $a^p - a$ per p diuidi poteſt; tum per idem p quoque formula $(a+1)^p - a - 1$ diuidi poterit.

Demonſtratio.

Reſoluatur $(1+a)^p$ conſueto more in ſeriem,

From Leonhard Euler's *Theorematum quorundam ad numeros primos spectantium demonstratio* (translation: *Certain proofs involving prime numbers*), written in 1736.

overview

How do we create mathematical masterpieces?

What sets apart a mathematical masterpiece from an ordinary mathematical result? How do you create new mathematics? Is all of mathematics already known? What does it even mean to "know" something in mathematics? What does it take to be mathematically creative?

This semester, we will explore some of the most beautiful and masterful results in mathematics, including results about numbers, the nature of infinity, and the unexpected relationships between geometry and polynomials. Along the way we will also address more fundamental questions like the ones above.

As you look at others' mathematical masterpieces, you will work to create some yourself. In particular, we will work on developing your ability to communicate with a modern mathematical audience using the language and customs of our field. Along with developing your ability to make a deductive argument about math, you will find that your ability to make arguments about everything from politics to restaurant choices will improve too!

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essential info

MAT246 LEC0101, Concepts in Abstract Mathematics
Prof. Sarah Mayes (smt@math.toronto.edu)
Fall 2021, q.utoronto.ca
$26 course cost for Top Hat subscription

learning goals

Everything that you are asked to do in MAT246 is motivated by **learning goals:** what you are expected to do upon completion of the course. By the end of this course you should be able to...

1. KNOW the material of this course at a proficient level . This means memorizing vital information so that you have it at their fingertips, being familiar enough with important information that you can easily find it when needed, and know that other results exist.
2. DEDUCE to understand and construct arguments in mathematics related to this material. You should refine your ability to use standard mathematical proof techniques, including induction and proof by contradiction, and identify errors in other arguments.
3. CREATE new and innovative mathematical proofs yourself, without simply mimicking a similar proof.
4. COMMUNICATE mathematical arguments clearly, both orally and in writing.
5. REFLECT on your learning and engage in reflection on the class.
6. CRITIQUE the written proofs of others and provide useful feedback to them.
7. BUILD COMMUNITY within the course and within the math department at the University of Toronto by being an active participant in classes and activities.

Major Content Milestones:

- Modular arithmetic: how does stripping away detail allow patterns to come alive?
- Cryptography: how can math be used to send and receive secret messages, even among people who have never exchanged a secret key?
- Number systems: what do we do when old number systems no longer suit our needs?
- Infinite sets: when is one infinity bigger than another?
- Polynomials & Geometry: what do polynomials have to do with trisecting angles?

Math Values:

Each week we'll highlight one "math value": something about the field of mathematics as a whole or the practice of doing math. These are lessons that you can take across all of your math classes at UofT and beyond.

Hard work and persistence are more important for success in math than natural ability.

- Maria Klawe, mathematician

Participants at the 1897 international congress of mathematicians (ICM) and the international mathematical union (which organizes the ICM) at the most recent ICM in Brazil.

activities & assignments, part 1

MAT246 assignments are designed so that you will parallel important mathematical actions: creating, deducing, knowing, communicating, analyzing, and building community. The assignments will work together with each other and with classes and tutorials to provide you with a solid basis for the course: some assessments are for learning ("formative") while other assessments are to show what you have learned at the end of a learning process ("summative"). Some assessments will be completed during tutorials, lectures, or specially scheduled sessions ("in class") while others will be completed on your own time ("out of class"). Some assessments will contribute to your final grade in the course ("graded") while others will not ("ungraded").

A more comprehensive summary of the assignments is linked on the course website. In brief, the *graded* assessments for the course are the following. I've paired each with some of the learning questions that this assignment helps answer.

Learning Material Annotation (formative, out of class, graded)

What we know: Reading math texts is tough and it is a skill that must be developed. Mathematicians read texts much more slowly than students and go back and forth as they read. Keeping up with classes is difficult in the midst of other commitments.

Questions this assessment addresses: How do you ensure that you keep up with the material in a class? How do you engage in intellectual conversations with classmates? How can we provide opportunities to read the text *closely* and reflect on what you've read?

Description: You will annotate texts and videos in Perusall and, in doing so, engage in a conversation with your classmate colleagues about what you are learning. You will have the opportunity to tag questions for the instructor to be answered in class.

activities & assignments,part 2

In-Class Voting (formative, in-class, graded)

What we know: Using active learning techniques like classroom voting is one of the best ways to encourage more equitable STEM practices. Retrieval practice is essential to form brain pathways for remembering.

Questions this assessment addresses: How can we make sure that we hear from all voices in a class, equally? How can we hear from students despite masks? How can we set up our classroom for a smooth transition online, if necessary?

Description: During lectures you will actively participate in what is happening. One way to provide your input and help both me and your peers know what you are thinking is through voting. We will use the responses system Top Hat. You will also submit your reflection question responses through Top Hat.

Get Involved! (formative, out-of-class, graded)

What we know: Students who know other students in their classes and who have "near-peer" role models in their majors are more likely to persist in a STEM major, like math. Being a math major is about more than just the classes you take!

Questions this assessment addresses: How can we encourage you to meet new people in and out of your classes (perhaps even give you an excuse to meet new people!)? How can we encourage you to take the time to attend math department talks in the midst of a busy schedule? How can we help you feel welcomed to attend math department talks?

Description: During the term you'll do some activities to meet new people in the math department or in your classes. The number of things that you do is based on a "points" system.

Quizzes (formative, out-of-class, ungraded)

What we know: Providing opportunities to self-check your progress can provide important feedback.

Questions this assessment addresses How are you progressing in your knowledge and deduction skills in the course? Are you "keeping up" with the material?

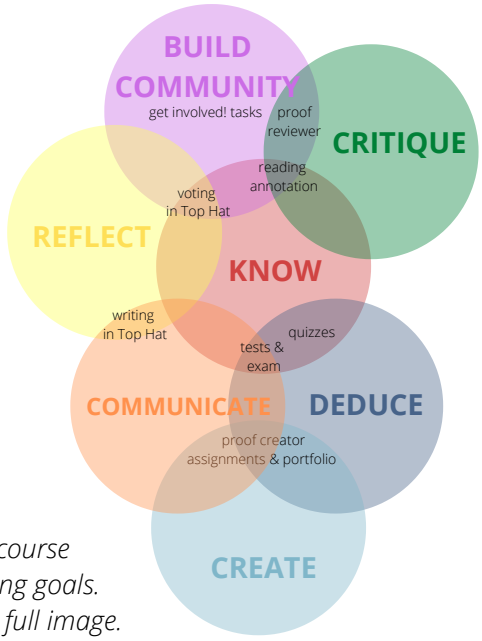
Description: We will post frequent online quizzes that can be used to self-check your progress in the course. The questions will be similar to multiple choice questions from tests & exams.

2 Tests & 1 Exam (summative, out-of-class, graded)

What we know: In order to learn math you need to do math. When preparing for tests & exams in math students do more practice than at any other time.

Questions this assessment addresses: Do you have the fundamental knowledge and deduction skills to demonstrate proficiency to earn this credit?

Description: These will be traditional, invigilated, closed-book assessments. You may be permitted to bring one 5-inch by 7-inch notecard blank on one side, with whatever you would like written on the other.



The assignments in this course integrate each of the learning goals. See the course website for the full image.

activities & assignments, part 3

Proof Creator (summative, out-of-class, partially graded)

What we know: Proof writing is creative and is at the core of what doing math is. It is a skill that can be developed, but needs lots of time and feedback.

Questions this assessment addresses: What is mathematical creativity? How do you tailor your argument to an audience? How can you develop your proof-writing skills?

Description: In this assignment you'll write proofs and have opportunities for TA and peer review before submitting them as part of a final Proof Portfolio. In your Proof Portfolio, you'll exhibit examples of proofs that satisfy different criteria and summarize what you've learned in your own CREATIVE way.

Proof Reviewer (summative, in class, graded)

What we know: Peer review is an important part of being a mathematician. It helps students build investment and skill in writing, and benefits "strong" students too. But students at your level don't provide good reviews when working alone; their reviews are much stronger when they work with a group.

Questions this assessment addresses: Is the logic in a mathematical argument correct? Are there multiple ways to prove a result that I've thought about? How can I built solid collaborations?

Description: During tutorials you will work in groups to provide peer reviews of other students' Proof Creator assignments.

MON	do pre-class reading & annotation on Perusall (KNOW & ARGUE)	TUTORIAL
TUE		1 hour tutorial on MON, WED, or FRI. Focus on CREATE, COMMUNICATE, & ANALYZE (many weeks components of Proof Creator project due)
WED	**2 hour class** (includes a technical workshop - CREATE, ARGUE - class feedback, and REFLECT)	
THU	Proof Creator project	
FRI	**50 minute class** (includes a focus on one proof - ANALYZE - and modern extension)	
SAT	review week's work & plan for *Get Involved!* tasks next week!	
SUN		

a typical week in MAT246

accommodations

If you require academic accommodations I am happy to work with you and your Accessibility Advisor to find a solution that helps you succeed in MAT246. Please fill-out the form here [ADD-insert link]. If you have a learning need requiring an accommodation, there are people at the University who can help!! Register at Accessibility Services at http://www.accessibility.utoronto.ca/Home.htm.

contacting us

Come to talk to us!! See "Office Hours" on Quercus

Questions? FAQ & Ed forum on Quercus

Confidential Questions? smt@math.toronto.edu
with "MAT246:" in title.

who are we?

I am Professor Sarah Mayes-Tang. I've been a Teaching-Stream Professor at the University for about 5 years, and prior to that I was a professor at Quest University Canada, a very small liberal arts university, for four years. I earned a PhD in mathematics from the University of Michigan with a dissertation entitled *Asymptotic Generic Initial Systems*, and a bachelor's degree in pure math from Queen's University.
As a Teaching Stream Professor I am interested in questions related to how students and teachers learn and how classes can run more effectively. I decided to focus on a career in education because throughout my PhD I found these questions just as complicated and deep as mathematical ones! I still do as much math research as possible. I hope to be able to tell you more about my background in both math and education during the semester.
While I will set the structure of this course you and your classmates are what will "make" it what it becomes. You bring unique and special things into the classroom. Please know that you are a vital part in every classroom that you walk into (virtual or physical!). You truly DO make a difference in the day of another student or TA or professor. It's sometimes hard to see that in a big classroom but you are near other people; I do see lots of students, even in front of a sea of them.

What should you call us?

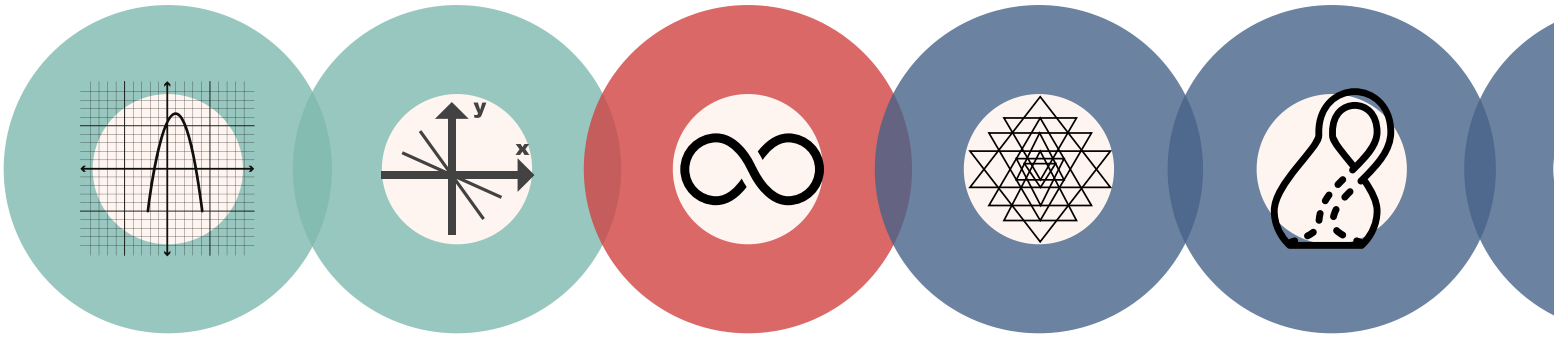
- Please call me "Professor Mayes-Tang" or "Dr. Mayes-Tang"
- You can call TAs by their first names
- Ask each other what they prefer to be called.
- I will "default" to using your first name or preferred name, but please correct me if I use the wrong name or wrong pronunciation.

Bourbaki Groups

This quote from Erdös captures his openness to collaboration: he knew that working with anyone on mathematics would result in new mathematical discoveries. Indeed, by this point in your mathematical careers you likely know that it is not the lonely pursuit that it is often portrayed. It is social. Some of you will know people in the class already. Others will not. I want to give you the opportunity - the grand and wonderful opportunity! - to meet new people and to do mathematics with them.
During the term you will work in consistent groups of about five of your classmates, called your Bourbaki groups. You will work with them in tutorial on your Proof Reviewer assignments, catch up with them if you need to miss a class, and review their final portfolios.

Another roof,
another proof.

- Paul Erdös, mathematician



Calculus | linear algebra | mat246: masterpieces! | group theory | topology...

PREREQUISITES FOR MAT246 AND SOME OF THE COURSES AFTER MAT246

the fine print.

THE COMPLETE SYLLABUS - with all fine print - is at the website on q.utoronto.ca. See the class website for:

- Links to the live streaming audio of the first two weeks of classes
- Minimal technical requirements for the course
- Technology set-up guide - programs and accounts you'll need for the course
- Tips for How to Succeed in MAT246
- Expectations for Attendance at Lectures (this is something that may change, based on University policy)
- Policy for submitting late work
- Appropriate use of laptops and cell phones during class time
- Online discussion forum guidelines
- Academic Integrity guidelines and the use of the University's plagiarism detection tool.

SUBMITTING WORK & RECORDING

- All work will be submitted electronically, online. Specific formatting information will be included in assignment instructions
- You may not record lectures without my written permission. You may not copy, email, or post, materials without getting my explicit permission beforehand. Otherwise this is an infringement of copyright and is prohibited. ;

REQUIRED TEXTS

- All Required Texts are available online. They will be posted on Perusall.
- The main text for the course is *A Readable Introduction to Real Mathematics by* Rosenthal^3. It is available through library.utoronto.ca

ED FORUM & EMAIL RESPONSE TIMES

- The Teaching Team will monitor the Ed Forum and aim to respond to unanswered questions within 24 hours during the week.
- I respond to email Monday-Friday only. I aim to respond to emails from students within 48 hours. I can only respond to emails that are confidential; other questions should be asked in the Ed Forum.

A SELECTION OF KEY DATES

date	event
September 23	Classes in-person
Week of Sept 27	First tutorials
October 5	Problem Set 1 Due
Friday, Oct 15	Test 1
October 26	Problem Set 2 Due
Friday, Nov 5	Test 2 (tentative)
November 23	Problem Set 3 Due
December 7	Portfolio Due