Critical Incident Questionnaire:

https://tinyurl.com/Unit1CIQ

If you've done this, here's two challenging integrals (answers next week):

$$\int \sin(e^t)dt$$

$$\int \sqrt{\tan(x)}dx$$

S7.1 – Integration Methods – Substitution

Assaf Bar-Natan

"You don't have to feel like a waste of space
You're original, cannot be replaced."

–"Firework", Katy Perry

Jan. 24, 2020

Reading Comprehension

The substitution technique tells us that if F is an antiderivative of f, then ______ is an antiderivative of $f(g)g'$.

Takeaway

When faced with an integral that has a function g inside another function, try a substitution.

Select all of the integrals where substitution could be used to evaluate the integral:

All results ▾

A	$\int x\sin(x^2)\,dx$	173
B	$\int x\sin(x)\,dx$	24
C	$\int x^2\sin(x)\,dx$	23
D	$\int (3x+2)(x^3+5x)^7\,dx$	34
E	$\int e^x\sqrt{1+e^x}\,dx$	164
F	$\int \frac{e^x-e^{-x}}{(e^x+e^{-x})^3}\,dx$	155
G	$\int \frac{\sin x}{x}\,dx$	23

Submissions Closed

If we are trying to evaluate the integral $\int e^{\cos\theta}\sin\theta\,d\theta$, which substitution would be most helpful?

✓ **91%** Answered Correctly

A	$u = \cos\theta$	138
B	$u = \sin\theta$	17
C	$u = e^{\cos\theta}$	29

Invalid date · Segment Results · Compare with session · **Show percentages** · **Hide Graph** · **Condense Text**

184/184 answered · Ask Again

Open · Closed · Responses · Correct · 100%

A Simple Substitution

Find an antiderivative, F of $\int e^{\cos\theta} \sin\theta d\theta$, with $F(0) = 0$.

A Simple Substitution

Find an antiderivative, F of $\int e^{\cos\theta}\sin\theta d\theta$, with $F(0) = 0$.
We substitute $\cos(\theta) = u$. Then $\frac{du}{d\theta} = -\sin(\theta)$. Thus,

$$\int e^{\cos\theta}\sin(\theta)d\theta = \int e^{u(\theta)}(-u'(\theta))d\theta = -e^{u(\theta)}$$

Thus, all of the antiderivatives of $e^{\cos\theta}\sin(\theta)$ are of the form $-e^{\cos\theta} + C$. To find the appropriate C, we plug in $\theta = 0$, and solve, to get:

$$F(\theta) = -e^{\cos\theta} + e$$

Submissions Closed

If we make the substitution $w = \ln x$, which of the following statements is true?

✓ **76%** Answered Correctly

A	$\displaystyle\int \frac{1}{x\ln x}dx = \int w\,dw.$	13
B	$\displaystyle\int \frac{1}{x\ln x}dx = \int \ln(w)\,dw.$	27
C	$\displaystyle\int \frac{1}{x\ln x}dx = \int \frac{1}{w}dw$	146
D	$\displaystyle\int \frac{1}{x\ln x}dx = \int \frac{1}{\ln(w)}dw$	6

Invalid date | Segment Results | Compare with session | Show percentages | Hide Graph | Condense Text

192/192 answered | Ask Again

Open | Closed | Responses | ✓ Correct | 100%

Substituting Back

Compute:

$$\int \frac{1}{x(\log(x))^2}$$

Where log is the natural logarithm.

Substituting Back

Compute:

$$\int \frac{1}{x(\log(x))^2}$$

Where log is the natural logarithm.

$$\int \frac{1}{x(\log(x))^2} dx = \frac{-1}{\log(x)} + C$$

We can verify this by differentiating.

Spot the Error

Blackie is sitting in the yard, lying on his back in the sun, computing integrals. That's just what cats do. He writes:

Spot the Error

Blackie is sitting in the yard, lying on his back in the sun, computing integrals. That's just what cats do. He writes:

To compute $\int_1^4 \sqrt{1+\sqrt{x}}dx$, I will let $w = 1+\sqrt{x}$, so $\frac{dw}{dx} = \frac{1}{2\sqrt{x}}$. Thus,

$$dx = dw(2\sqrt{x}) = dw(2(w-1))$$

Plugging this in, I get:

$$\int_1^4 \sqrt{1+\sqrt{x}}dx = \int_1^4 \sqrt{w}(2(w-1))dw$$
$$= \int_1^4 (2w^{3/2} - 2w^{1/2})dw$$
$$= \left[2\frac{2}{5}w^{5/2} - 2\frac{2}{3}w^{3/2}\right]_1^4$$

Takeaway

When substituting in a definite integral, don't forget to change your bounds!

Lexi and Obie and Mouse

Lexi, the tail-less cat, is meowing at Obie for stealing her mouse. As her mouth opens, the size, S, of the opening changes as a function of time: $S = g(t)$.

Lexi and Obie and Mouse

Lexi, the tail-less cat, is meowing at Obie for stealing her mouse. As her mouth opens, the size, S, of the opening changes as a function of time: $S = g(t)$.
Let m be the volume of the meow. Denote by $\frac{dm}{dS} = f(S)$, and let Δm be the change in meow volume between $1s$ and $2s$.

Lexi and Obie and Mouse

Lexi, the tail-less cat, is meowing at Obie for stealing her mouse. As her mouth opens, the size, S, of the opening changes as a function of time: $S = g(t)$.

Let m be the volume of the meow. Denote by $\frac{dm}{dS} = f(S)$, and let Δm be the change in meow volume between $1s$ and $2s$.

Fill in the following:

$$\Delta m = \int_{\square}^{\square} f(g(t))g'(t)dt$$

$$\Delta m = \int_{\square}^{\square} f(s)ds$$

$$\Delta m = \int_{\square}^{\square} dm$$

Lexi and Obie and Mouse

$$\Delta m = \int_1^2 f(g(t))g'(t)dt$$

$$\Delta m = \int_{g(1)}^{g(2)} f(s)ds$$

$$\Delta m = \int_{m(g(1))}^{m(g(2))} dm$$

Plans for the Future

For next time:

WeBWork 7.2 and read section 7.2