

**Critical Incident Questionnaire:**  
<https://tinyurl.com/Unit1CIQ>

If you've done this, here's two challenging integrals (answers next week):

$$\int \sin(e^t) dt$$
$$\int \sqrt{\tan(x)} dx$$



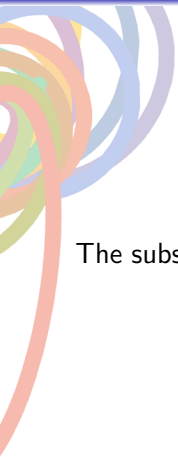
## S7.1 – Integration Methods – Substitution

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“You don’t have to feel like a waste of space  
You’re original, cannot be replaced.”

– “Firework”, Katy Perry

Jan. 24, 2020



The substitution technique tells us that if  $F$  is an antiderivative of  $f$ , then \_\_\_\_\_ is an antiderivative of  $f(g)g'$ .

# Takeaway



**When faced with an integral that has a function  $g$  inside another function, try a substitution.**

Select all of the integrals where substitution could be used to evaluate the integral:

All results ▾

A	$\int x \sin(x^2) dx$	<input checked="" type="checkbox"/>	173
B	$\int x \sin(x) dx$	<input checked="" type="checkbox"/>	24
C	$\int x^2 \sin(x) dx$	<input checked="" type="checkbox"/>	23
D	$\int (3x + 2)(x^3 + 5x)^7 dx$	<input checked="" type="checkbox"/>	34
E	$\int e^x \sqrt{1 + e^x} dx$	<input checked="" type="checkbox"/>	164
F	$\int \frac{e^x - e^{-x}}{(e^x + e^{-x})^3} dx$	<input checked="" type="checkbox"/>	155
G	$\int \frac{\sin x}{x} dx$	<input checked="" type="checkbox"/>	23

Submissions Closed

If we are trying to evaluate the integral  $\int e^{\cos \theta} \sin \theta d\theta$ , which substitution would be most helpful?

✓ 91% Answered Correctly

A  $u = \cos \theta$



138

B  $u = \sin \theta$



17

C  $u = e^{\cos \theta}$



29

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184/184 answered

Ask Again

Open Closed Responses Correct

Q 100%

# A Simple Substitution



Find an antiderivative,  $F$  of  $\int e^{\cos \theta} \sin \theta d\theta$ , with  $F(0) = 0$ .

# A Simple Substitution

Find an antiderivative,  $F$  of  $\int e^{\cos\theta} \sin\theta d\theta$ , with  $F(0) = 0$ .  
We substitute  $\cos(\theta) = u$ . Then  $\frac{du}{d\theta} = -\sin(\theta)$ . Thus,

$$\int e^{\cos\theta} \sin(\theta) d\theta = \int e^{u(\theta)} (-u'(\theta)) d\theta = -e^{u(\theta)}$$

Thus, all of the antiderivatives of  $e^{\cos\theta} \sin(\theta)$  are of the form  $-e^{\cos\theta} + C$ . To find the appropriate  $C$ , we plug in  $\theta = 0$ , and solve, to get:

$$F(\theta) = -e^{\cos\theta} + e$$





Submissions Closed

If we make the substitution  $w = \ln x$ , which of the following statements is true?

✓ 76% Answered Correctly

A	$\int \frac{1}{x \ln x} dx = \int w dw.$		13
B	$\int \frac{1}{x \ln x} dx = \int \ln(w) dw.$		27
C	$\int \frac{1}{x \ln x} dx = \int \frac{1}{w} dw$		146
D	$\int \frac{1}{x \ln x} dx = \int \frac{1}{\ln(w)} dw$		6

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192/192 answered

Ask Again

⏪ ⏩ 🔍 Open 🔒 Closed 📄 Responses ✓ Correct ⏭

Q 100% 🏠

# Substituting Back

Compute:

$$\int \frac{1}{x(\log(x))^2}$$

Where  $\log$  is the natural logarithm.

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Compute:


$$\int \frac{1}{x(\log(x))^2}$$

Where  $\log$  is the natural logarithm.

$$\int \frac{1}{x(\log(x))^2} dx = \frac{-1}{\log(x)} + C$$

We can verify this by differentiating.

# Spot the Error



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To compute  $\int_1^4 \sqrt{1 + \sqrt{x}} dx$ , I will let  $w = 1 + \sqrt{x}$ , so  $\frac{dw}{dx} = \frac{1}{2\sqrt{x}}$ . Thus,

$$dx = dw(2\sqrt{x}) = dw(2(w - 1))$$


Plugging this in, I get:

$$\begin{aligned} \int_1^4 \sqrt{1 + \sqrt{x}} dx &= \int_1^4 \sqrt{w}(2(w - 1))dw \\ &= \int_1^4 (2w^{3/2} - 2w^{1/2})dw \\ &= \left[ 2\frac{2}{5}w^{5/2} - 2\frac{2}{3}w^{3/2} \right]_1^4 \end{aligned}$$



**When substituting in a definite integral, don't forget to change your bounds!**

# Lexi and Obie and Mouse



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Let  $m$  be the volume of the meow. Denote by  $\frac{dm}{dS} = f(S)$ , and let  $\Delta m$  be the change in meow volume between  $1s$  and  $2s$ .



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Fill in the following:

$$\Delta m = \int_{\square}^{\square} f(g(t))g'(t)dt$$

$$\Delta m = \int_{\square}^{\square} f(s)ds$$

$$\Delta m = \int_{\square}^{\square} dm$$


$$\Delta m = \int_1^2 f(g(t))g'(t)dt$$

$$\Delta m = \int_{g(1)}^{g(2)} f(s)ds$$

$$\Delta m = \int_{m(g(1))}^{m(g(2))} dm$$



For next time:

**WeBWork 7.2 and read section 7.2**