S6.3 – Differential Equations and Motion

Assaf Bar-Natan

"'Cause you can't stop the motion of the ocean or the sun in the sky
You can wonder, if you wanna, but I never ask why
And if you try to hold me down, I'm gonna spit in your eye and say
That you can't stop the beat! "

–" You can't Stop The Beat ", Hairspray

Jan. 20, 2020

Example: The S.I.R Model of Infection

The cats are getting sick. Let t be the time, in days, since the illness outbreak, and let:

- N be the total number of cats
- $S(t)$ be the number of cats susceptible to the disease
- $I(t)$ be the number of cats infected with the disease
- $R(t)$ be the number of cats who recovered from the disease

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The S.I.R model says that I, S, and R satisfy:

$$\frac{dS}{dt} = -\beta\frac{I(t)S(t)}{N}$$
$$\frac{dI}{dt} = \beta\frac{I(t)S(t)}{N} - \gamma I(t)$$
$$\frac{dR}{dt} = \gamma I(t)$$

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Use these equations to show that $\frac{dS}{dt} + \frac{dI}{dt} + \frac{dR}{t} = 0$. What does this tell us about $S + I + R$?

Takeaway

Differential equations appear in unlikely places, and their solutions have important real-world reprecussions.

Submissions Closed

For the differential equation $\frac{dy}{dx} = 5$, what is the most general family of functions that solves it?

A Constant

B Linear

C Polynomial

D Exponential (or vertically-shifted exponential)

195/196 answered

Ask Again

Open | Closed | Responses | ✓ Correct

100%

Submissions Closed

For the differential equation $\frac{dy}{dx} = 5x$, what is the most general family of functions that solves it?

- **A** Constant
- **B** Linear
- **C** Polynomial
- **D** Exponential (or vertically-shifted exponential)

191/191 answered

Ask Again

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Submissions Closed

For the differential equation $\frac{dy}{dx} = 5y$, what is the most general family of functions that solves it?

A Constant

B Linear

C Polynomial

D Exponential

208/208 answered

Ask Again

Open Closed Responses Correct

100%

Submissions Closed

For the differential equation $\frac{dy}{dx} = 0$, what is the most general family of functions that solves it?

A Constant

B Linear

C Polynomial

D Exponential (or vertically-shifted exponential)

197/197 answered Ask Again

Open Closed Responses Correct 100%

Cats Jumping

Roy sneaks up on Blackie, and surprises him with a loud meow. Blackie jumps straight into the air at a speed of $3m/s$.

1 min. Write a differential equation that involves Blackie's velocity (in m/s) while he's in the air.

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1 min. What is the appropriate constant to choose? $C = 3$ because $v(0) = 3m/s$

Submissions Closed

If two solutions to $\frac{dy}{dx} = f(x)$ have different values at $x = 3$ then they have different values at every x.

A True, and I am confident in my answer.

B True, and I am not confident in my answer.

C False, and I am not confident in my answer.

D False, and I am confident in my answer.

207/208 answered

Ask Again

Open | Closed | Responses | Correct

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Roy sneaks up on Blackie, and surprises him with a loud meow. Blackie jumps straight into the air at a speed of $3m/s$. We know that Blackie's velocity, $v(t) = 3 - 9.8t$, measured in m/s.

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1 min. What is the appropriate constant to choose? $D = 0$ because Blackie starts on the ground.

Submissions Closed

We've just seen that if acceleration is constant, then the position is a quadratic function of time. Is the reverse true? That is, if position is a quadratic function of time, then acceleration is constant

- **A** True, and I can prove it.
- **B** True, and I am not sure how to prove it.
- **C** False, but I'm not sure why.
- **D** False, and I have a counter-example.

203/204 answered   Ask Again

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Spend one minute writing a list of steps (from the start of the question to its finish) outlining how you could compute how high Blackie jumps.

PCats Jumping – The Steps

- Read the question
- Write the differential equation
- Find a family of solutions to the differential equation
- Find the right constants, and narrow down the family to one function
- Repeat the last three steps until we have the desired function (in our case, it was the height function)
- Optimize

Plans for the Future

For next time:
WeBWork 6.4 and read section 6.4