



S6.3 – Differential Equations and Motion

Assaf Bar-Natan

“Cause you can't stop the motion of the ocean or the sun in the sky
You can wonder, if you wanna, but I never ask why
And if you try to hold me down, I'm gonna spit in your eye and say
That you can't stop the beat! ”

–“ You can't Stop The Beat ”, Hairspray

Jan. 20, 2020

Example: The S.I.R Model of Infection

The cats are getting sick. Let t be the time, in days, since the illness outbreak, and let:

- N be the total number of cats
- $S(t)$ be the number of cats susceptible to the disease
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- $R(t)$ be the number of cats who recovered from the disease

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The S.I.R model says that I , S , and R satisfy:

$$\begin{aligned}\frac{dS}{dt} &= -\beta \frac{I(t)S(t)}{N} \\ \frac{dI}{dt} &= \beta \frac{I(t)S(t)}{N} - \gamma I(t) \\ \frac{dR}{dt} &= \gamma I(t)\end{aligned}$$

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Very difficult goal: Find the functions S , I , and R
Use these equations to show that $\frac{dS}{dt} + \frac{dI}{dt} + \frac{dR}{dt} = 0$. What does this tell us about $S + I + R$?



Differential equations appear in unlikely places, and their solutions have important real-world repercussions.









 Submissions Closed

For the differential equation $\frac{dy}{dx} = 5$, what is the most general family of functions that solves it?

- A Constant
- B Linear
- C Polynomial
- D Exponential (or vertically-shifted exponential)

195/196 answered

[Ask Again](#)

      Responses  **Correct** 

 100% 



Submissions Closed

For the differential equation $\frac{dy}{dx} = 5x$, what is the most general family of functions that solves it?

- A Constant
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191/191 answered

[Ask Again](#)



Responses



Q 100%





Submissions Closed

For the differential equation $\frac{dy}{dx} = 5y$, what is the most general family of functions that solves it?

A Constant

B Linear

C Polynomial

D Exponential

208/208 answered

[Ask Again](#)



Responses



Q 100%





Submissions Closed

For the differential equation $\frac{dy}{dx} = 0$, what is the most general family of functions that solves it?

A Constant

B Linear

C Polynomial

D Exponential (or vertically-shifted exponential)

197/197 answered

[Ask Again](#)



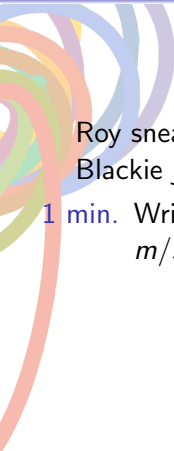
Responses



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Cats Jumping



Roy sneaks up on Blackie, and surprises him with a loud meow.
Blackie jumps straight into the air at a speed of $3m/s$.

- 1 min. Write a differential equation that involves Blackie's velocity (in m/s) while he's in the air.

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- 1 min. What is the appropriate constant to choose? $C = 3$ because
 $v(0) = 3m/s$



Submissions Closed

If two solutions to $\frac{dy}{dx} = f(x)$ have different values at $x = 3$ then they have different values at every x .

- A True, and I am confident in my answer.
- B True, and I am not confident in my answer.
- C False, and I am not confident in my answer.
- D False, and I am confident in my answer.

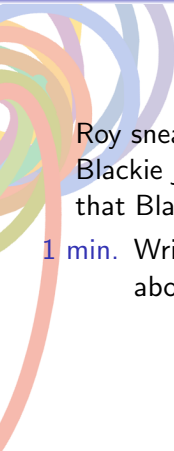
207/208 answered

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⏪ ⏩ ⏴ ⏵ 🔍 Open **🔒 Closed** 📄 Responses ✓ Correct ⏭

🔍 100% 🏠

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Roy sneaks up on Blackie, and surprises him with a loud meow. Blackie jumps straight into the air at a speed of $3m/s$. We know that Blackie's velocity, $v(t) = 3 - 9.8t$, measured in m/s .

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- 1 min. What is the appropriate constant to choose? $D = 0$ because Blackie starts on the ground.



Submissions Closed

We've just seen that if acceleration is constant, then the position is a quadratic function of time. Is the reverse true? That is, if position is a quadratic function of time, then acceleration is constant

- A True, and I can prove it.
- B True, and I am not sure how to prove it.
- C False, but I'm not sure why.
- D False, and I have a counter-example.

203/204 answered

[Ask Again](#)

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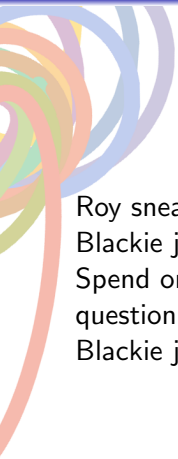
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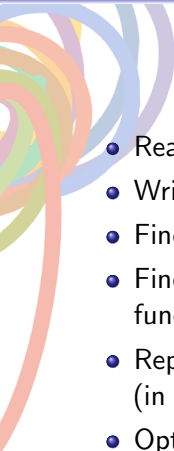
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PCats Jumping – The Steps

- 
- Read the question
 - Write the differential equation
 - Find a family of solutions to the differential equation
 - Find the right constants, and narrow down the family to one function
 - Repeat the last three steps until we have the desired function (in our case, it was the height function)
 - Optimize

Plans for the Future



For next time:

WeBWork 6.4 and read section 6.4