S6.1 – Analyzing Antiderivatives Algebraically

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" Now the teacher would say to learn your algebra
But I'd bring home C's and D's
How could I make an A when there's a swingin' maid
On the left and on the right and in the back and the front of me? "

–" Straight A's in Love ", Johnny Cash

Jan. 17, 2020

WeBWork Reflection

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- Share with your group your progress or how you solved it.

Takeaway

MAT136 tip: WeBWork questions are hard! Help each other!

Submissions Closed

What type of object is each of the following 'integrals'?

Premise		Response	
1	$\int_t^3 f(x)dx$	A	function of t
2	$\int_\pi^{100} g(t)dt$	B	infinite family of functions
3	$\int 2dx$	C	function of x
4	$\int_1^x h(t)dt$	D	number

202/202 answered

Ask Again

Open | Closed | Responses | Correct

100%

Cats and Hay-Bales

The cats (Marzipan, Obie, Blackie, and Roy) are cuddling up in a carved out hay bale. Let t be the time, in minutes, that the cats spend in the cavity. They heat up the cavity at a rate of $3e^{-0.2t}$ degrees Celsius per minute. After six minutes, the temperature was measured to be $13^\circ C$.

- Write an expression for the temperature two minutes after the cats jumped into the cavity.

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- Write an expression for the temperature two minutes after the cats jumped into the cavity.
- Find the antiderivative of $3e^{-0.2t}$.
- What was the temperature when $t = 2$?

Solution

- $T(2) = 13 - \int_2^6 3e^{-0.2t}\,dt$
- $\int 3e^{-0.2t}\,dt = 3\int e^{-0.2t}\,dt = \frac{3e^{-0.2t}}{-0.2}$

Takeaway

For any function, f, and a co-ordinate (x, y), there is a single antiderivative F, for which $F(x) = y$.

Submissions Closed

If F and G are antiderivatives of f, then $F - G$ is an antiderivative of

A f

B $2f$

C Any constant

D 0

208/209 answered

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Punctuated Lecture – Finding All Antiderivatives

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This means that $F - G$ is constant. **Why?**

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What does this tell us about any other antiderivative of f?

Cats and Logs

Mia and Obie are having a fight. Both want to compute $\int \frac{1}{5x}$.

Mia says:	Obie says:
"I can pull out $\frac{1}{5}$, and use $$\frac{d}{dx}\log(\lvert x\rvert) = \frac{1}{x}$$ to get that every antiderivative of $\frac{1}{5x}$ is of the form $\frac{1}{5}\log(\lvert x\rvert) + C$."	"When I compute the derivative of $\frac{1}{5}\log(\pi\lvert x\rvert)$, I get $\frac{1}{5x}$, so $$\frac{1}{5}\log(\pi\lvert x\rvert)$$ is an antiderivative of $\frac{1}{5x}$ that doesn't fit your pattern."

Who is right?

Solution

Both are right, because if we apply logarithm rules, we get:

$$\frac{1}{5}\log(\pi|x|) = \frac{1}{5}\log(|x|) + \frac{1}{5}\log(\pi)$$

which is of the form that Mia wanted.

Plans for the Future

For next time:

WeBWork 6.3 and read section 6.3