## S6.1 - New Technology - Antiderivatives

## Assaf Bar-Natan (Replacing Josh Lackman)

" They took the credit for your second symphony
Rewritten by machine on new technology
And now I understand the problems you can see Oh, ah, oh! "
-" Video Killed the Radio Star ", The Buggles
Jan. 16, 2020

## The Definition of an Antiderivative

If $f$ and $F$ are two functions, we say that $F$ is an antiderivative of

$$
f \text { if } F^{\prime}(x)=f(x) .
$$

## The Definition of an Antiderivative

If $f$ and $F$ are two functions, we say that $F$ is an antiderivative of

$$
f \text { if } F^{\prime}(x)=f(x) .
$$

For example: if $f(x)=2 x$ and $F(x)=x^{2}$, then $F(x)$ is an antiderivative of $f(x)$.

If $F(x)$ and $G(x)$ are antiderivatives of a function $f(x)$, then $H(x)=F(x)+G(x)$ is also an antiderivative of $\mathrm{f}(\mathrm{x})$

A True, and I am confident in my answer.
B True, and I am not confident in my answer.
C False, and I am not confident in my answer.
D False, and I am confident in my answer.

## Takeaway

## MAT136 tip: When you know the definition, use it instead of taking shortcuts.

## Draw The Antiderivative

- Take out a sheet of paper, or borrow one from your neighbour.


## Draw The Antiderivative

- Take out a sheet of paper, or borrow one from your neighbour.

Draw a continuous function defined on $[0,5]$ which is:
(1) Decreasing and linear on $[0,2]$.
(2) Positive at 0 and negative at 2
(3) Equal to a positive constant between 4 and 5 .

Make sure your axes are labelled!

## Draw The Antiderivative

- Take out a sheet of paper, or borrow one from your neighbour.

Draw a continuous function defined on $[0,5]$ which is:
(1) Decreasing and linear on $[0,2]$.
(2) Positive at 0 and negative at 2
(3) Equal to a positive constant between 4 and 5 .

Make sure your axes are labelled!

- Find a partner, and exchange your papers


## Draw The Antiderivative

- Take out a sheet of paper, or borrow one from your neighbour.
- Draw a continuous function defined on $[0,5]$ which is:
(1) Decreasing and linear on $[0,2]$.
(2) Positive at 0 and negative at 2
(3) Equal to a positive constant between 4 and 5 .

Make sure your axes are labelled!

- Find a partner, and exchange your papers
- Draw the graph of an antiderivative of the function your partner drew.


## Draw The Antiderivative

- Take out a sheet of paper, or borrow one from your neighbour.
- Draw a continuous function defined on $[0,5]$ which is:
(1) Decreasing and linear on $[0,2]$.
(2) Positive at 0 and negative at 2
(3) Equal to a positive constant between 4 and 5 .

Make sure your axes are labelled!

- Find a partner, and exchange your papers
- Draw the graph of an antiderivative of the function your partner drew.
- Pass the papers back to your partner, and compare your answers. Explain what you drew.


## Draw The Antiderivative

- Take out a sheet of paper, or borrow one from your neighbour.
- Draw a continuous function defined on $[0,5]$ which is:
(1) Decreasing and linear on $[0,2]$.
(2) Positive at 0 and negative at 2
(3) Equal to a positive constant between 4 and 5 .

Make sure your axes are labelled!

- Find a partner, and exchange your papers
- Draw the graph of an antiderivative of the function your partner drew.
- Pass the papers back to your partner, and compare your answers. Explain what you drew.
- With your partner, pick a drawing, and draw on it an antiderivative of the original function that is different from the one you already drew


## Draw The Antiderivative - My Drawing



## Draw The Antiderivative - My Drawing



## Draw The Antiderivative - My Drawing



## Takeaway

If $F$ is an antiderivative of $f$, then $F+c$ is an antiderivative of $f$ for any constant $c$

## Summarizing What We Know

| Feature of function at a <br> point | Feature of an antideriva- <br> tive at that point |
| :--- | :--- |
| positive |  |
| negative |  |
| $x$-intercept |  |
| increasing |  |
| decreasing |  |
| maximum |  |
| minimum |  |

## Summarizing What We Know

| Feature of function at a <br> point | Feature of an antideriva- <br> tive at that point |
| :--- | :--- |
| positive | increasing |
| negative | decreasing |
| $x$-intercept | concave up |
| increasing | concave down |
| decreasing | inflection point |
| maximum | inflection point |
| minimum |  |

## Takeaway

In the same way that we sketch a function's derivative, we can reverse the process to sketch the antiderivative.

## Antiderivatives and the F.T.C

Recall that if $F$ is a differentiable function on an interval $[a, b]$, and $F^{\prime}=f$, then:

$$
\int_{a}^{b} f(x) d x=F(b)-F(a)
$$

## Antiderivatives and the F.T.C

Recall that if $F$ is a differentiable function on an interval $[a, b]$, and $F^{\prime}=f$, then:

$$
\int_{a}^{b} f(x) d x=F(b)-F(a)
$$

Knowing the antiderivative allows us to compute definite integrals easily.

The cats are cuddling up in a carved out hay bale. Let $t$ be the time, in minutes, that the cats spend in the cavity. They heat up the cavity at a rate of $r(t)$ degrees
Celsius per minute. Knowing $\boldsymbol{r}(\boldsymbol{t})$ for all $t$ between 0 and 6 is enough information to determine the temperature of the cavity at $t=6$

A True, and I know how to compute it.
B True, but I'm not sure why.
C False, but I can't explain why I think this.
D False, and I know what information is missing.

The cats are cuddling up in a carved out hay bale. Let $t$ be the time, in minutes, that the cats spend in the cavity. They heat up the cavity at a rate of $r(t)$ degrees Celsius per minute. After six minutes, the temperature was measured to be $13^{\circ} \mathrm{C}$. What is a formula that describes the temperature at $\mathrm{t}=0$ ?

A $\int_{6}^{0} r(t) d t+13$
B $\int_{0}^{6} r(t) d t-13$
c $\int_{0}^{6} r(t) d t+13$
D $\int_{6}^{0} r(t) d t-13$
$0 / 10$ answered


## Plans for the Future

## For next time:

## WeBWork 6.2 and read section 6.2

