

# Welcome to MAT135 LEC0501 (Assaf)



Share with your neighbour something you did during this rainy weekend.

Submissions Closed

Suppose that  $f$  is a continuous function. Then  $\int_0^2 f(x) dx = \int_0^2 f(t) dt$

✓ 77% Answered Correctly

A	True, and I am confident in my answer.	<div style="width: 77%;"></div>	85
B	True, and I am not confident in my answer.	<div style="width: 10%;"></div>	45
C	False, and I am not confident in my answer.	<div style="width: 3%;"></div>	17
D	False, and I am confident in my answer.	<div style="width: 3%;"></div>	22

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169/170 answered

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Q 100% [⌵](#)



## S5.4 – Properties, Theorems, and Bounds on Definite Integrals


Assaf Bar-Natan

“ On a tour of one-night stands my suitcase and guitar in hand  
And every stop is neatly planned for a poet and a one-man band...  
Homeward bound ”

–“ Homeward Bound ”, Simon & Garfunkel

Jan. 13, 2020

# Takeaway



**In expressions like  $\int_a^b f(x)dx$ , the variable  $x$  is a dummy variable – It only is there to remind us that  $f$  is a function and that we are integrating with respect to its input.**



**Get into groups of 3-4.**



## Get into groups of 3-4.

- Go around your group, and one by one state an integration theorem.



## Get into groups of 3-4.

- Go around your group, and one by one state an integration theorem.
- Go through the textbook, and make sure all of the theorems from chapter 5.4 have been stated.

# Draw a Theorem

Below is a summary of some of the theorems from chapter 5.4:

$$\int_a^b (f(x) + g(x))dx = \int_a^b f(x)dx + \int_a^b g(x)dx$$

$$\int_a^b cf(x)dx = c \int_a^b f(x)dx$$

$$\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$$

And some of the bounds:

$$m \leq f(x) \leq M \Rightarrow m(b - a) \leq \int_a^b f(x)dx \leq M(b - a)$$

$$f(x) \leq g(x) \Rightarrow \int_a^b f(x)dx \leq \int_a^b g(x)dx$$



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In your group, choose one of these theorems and one of these bounds, and draw a picture explaining why it's true.

 Submissions Closed

If  $\int_a^b f(x) dx \leq \int_a^b g(x) dx$  then on the interval  $[a,b]$ ,  $f(x) \leq g(x)$

✓ 63% Answered Correctly

<b>A</b>	True, and I can explain why	<div style="width: 20%; background-color: #00AEEF;"></div>	50
<b>B</b>	True, and I'm not sure why	<div style="width: 10%; background-color: #00AEEF;"></div>	29
<b>C</b>	False and I'm not sure why	<div style="width: 5%; background-color: #008000;"></div>	24
<b>D</b>	False, and I have a counter-example	<div style="width: 30%; background-color: #008000;"></div>	108

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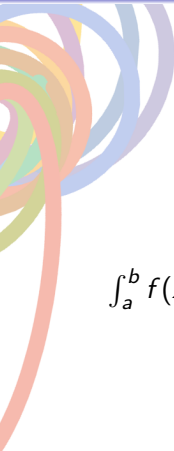
211/211 answered

[Ask Again](#)

[^](#) [<](#) [>](#) [Open](#) [Closed](#) [Responses](#) [Correct](#) [»](#)

[Q](#) 100% [⌵](#)

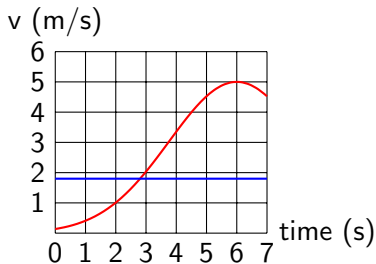
# Takeaway



**If we know that  $f(x) \leq g(x)$  on  $[a, b]$ , then  $\int_a^b f(x)dx \leq \int_a^b g(x)dx$ . However, we cannot reverse this!**

# An Application

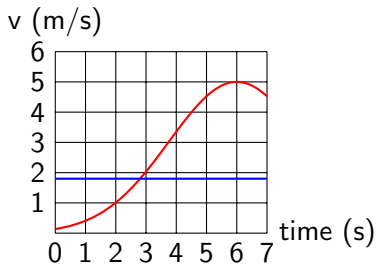
Marzipan is chasing a mouse along the side of the barn. The mouse has a head start of about  $1m$ , and the velocities of Marzipan (red) and the mouse (blue) are plotted below:



Will Marzipan catch the mouse?

# An Application

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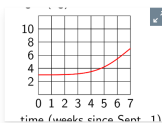


Will Marzipan catch the mouse? When?



Submissions Closed

Obie the cat is bulking up for the cold winter. His weight,  $w(t)$  is given by the red line in the graph. Which of the following statements are incorrect (select ALL incorrect statements)?



✓ 12% Answered Correctly

A	Obie's average weight in the last two weeks is more than his average weight over the entire seven weeks	<input type="checkbox"/>	57
B	Obie's average weight over the seven weeks is an increasing function	<input type="checkbox"/>	48
C	Obie's average weight over the seven weeks is equal to $w(7) - w(0)$	<input checked="" type="checkbox"/>	159
D	Obie's average weight over the seven weeks is somewhere between 8 and 3	<input type="checkbox"/>	54

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186/187 answered

Ask Again



Responses

Correct

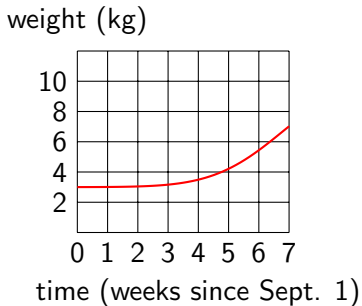


88%



# Using Integrals to Estimate Averages

Obie's weight over the fall season is plotted below:



Estimate Obie's average weight during this time.

# Plans for the Future



For next time:

**WeBWork 6.1 and read section 6.1**