Welcome to MAT136 LEC0501 (Assaf)

Critical Incident Questionnaire 3:
https://tinyurl.com/March2020CIQ

S10.3 – Taylor Series – Applications (Part 2)

Assaf Bar-Natan

"Everything will be alright, if
We just keep dancing like we're twenty-two..."

–"22", Taylor Swift

March 30, 2020

Taylor Series and Substitution

Recall:

If a Taylor series for $f(x)$ converges for x on some interval, then the Taylor series for $f(g(x))$ converges whenever $g(x)$ is in that interval

If a Taylor series for $f(x)$ converges for x on some interval, then the Taylor series for $f'(x)$ converges on the same interval

Submissions Closed

Determine the Radius of Convergence of each of the following series, by using Radii of convergence that you know.

Premise		→	Response	
1	$4x \cdot e^x \cos(x)$	→	A	∞ by substitution of polynomials into known Taylor Series
2	$\dfrac{1}{1-(x/4)}$	→	B	4 by differentiation of known Taylor series
3	$\dfrac{1}{4(1-(x/4))^2}$	→	C	4 by substitution of polynomials into known Taylor Series
4	$\cos(x^2+4x^3)$	→	D	∞ by multiplication of known Taylor series and polynomials

141/141 answered   Ask Again

Open | Closed | Responses | Correct   88%

Radius of Convergence

Consider $\frac{1}{1-(x/4)}$. The Taylor series around 0 is:

$$1 + y + y^2 + \cdots$$

Where $y = x/4$.
This converges when $-1 < y < 1$, ie, when $-4 < x < 4$, so the Taylor series converges on this interval by subtituting $x/4$ into a known Taylor series.

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We know that converges when $-4 < x < 4$, because it's the derivative of a Taylor series that converges on that interval.

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The Taylor series for $\cos(x)$ converges for any x, so no matter what we substitute into cos, the Taylor series will converge.

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If two functions have Taylor series which converge everywhere, then their product also has a Taylor series that converges everywhere

This is just an application of the product formula for Taylor series (Example 4)

Computing Series Using Derivatives

We are going to compute the series:

$$\sum_{n=1}^{\infty} \frac{3^{2n-1}2n}{n!}$$

- Identify constants, indices, and patterns in the series
- Substitute a variable into the series
- Interpret each term as an integral or derivative of something
- Integrate or differentiate term by term to get a new series
- Do you recognize the new series? If not, go back to step 2.
- Write the new series in closed form, and interpret the original series as its derivative or integral

Computing Series Using Derivatives

$$\sum_{n=1}^{\infty} \frac{3^{2n-1}2n}{n!}$$

The constants here are $2, 3$ (and 1). The index is n. We will try replacing all instances of 3 with the variable x:

$$\sum_{n=1}^{\infty} \frac{x^{2n-1}2n}{n!}$$

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$$\frac{x^{2n-1}2n}{n!} = \frac{d}{dx}\left(\frac{x^{2n}}{n!}\right)$$

Computing Series Using Derivatives

Remembering that we are evaluating when $x = 3$...

$$\sum_{n=1}^{\infty} \frac{x^{2n-1} 2n}{n!} = \sum_{n=1}^{\infty} \frac{d}{dx}\left(\frac{x^{2n}}{n!}\right) = \frac{d}{dx} \sum_{n=1}^{\infty} \left(\frac{x^{2n}}{n!}\right)$$

Q: Do you recognize this series?

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$$\sum_{n=1}^{\infty} \frac{x^{2n-1}2n}{n!} = \frac{d}{dx}\sum_{n=1}^{\infty}\left(\frac{x^{2n}}{n!}\right) = \frac{d}{dx}\left(e^{x^2} - 1\right)$$

Computing Series Using Derivatives

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$$\sum_{n=1}^{\infty} \frac{x^{2n-1} 2n}{n!} = \frac{d}{dx}\left(e^{x^2} - 1\right) = 2xe^{x^2}$$

Q: What is $\sum_{n=1}^{\infty} \frac{3^{2n-1} 2n}{n!}$?

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Q: What is $\sum_{n=1}^{\infty} \frac{3^{2n-1} 2n}{n!}$? We plug in $x = 3$ to get:

$$\sum_{n=1}^{\infty} \frac{3^{2n-1} 2n}{n!} = 6e^9$$

Submissions Closed

Find the exact sum of the series

$$\sum_{n=1}^{\infty} n(0.2)^{n-1}$$

44/44 answered

Ask Again

Open | Closed | Responses | Correct

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The Series $\sum n(0.2)^{n-1}$

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Interpret each term as a derivative to get:

$$\sum_{n=1}^{\infty} nx^{n-1} = \frac{d}{dx}\sum_{n=1}^{\infty} x^n = \frac{d}{dx}\left(\frac{1}{1-x} - 1\right)$$

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Everything worked because $|x| < 1$, so the series above converge

Takeaway

When we have a series, we can plug in a variable, x, then interpret it as a derivative or an integral of series that we know

Plans for the Future

For next time:

Watch the uploaded Section 10.3 video, and go over Taylor Solutions to ODEs