Welcome to MAT136 LEC0501 (Assaf)

Final exam information is on the main course website, under Test & Exam

S10.3 – Taylor Series – Applications

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"They'll tell you I'm insane
But I've got a blank space baby
And I'll write your name"

–"Blank Space", Taylor Swift

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Taylor Series and Substitution

Key observation: the equation

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A: We need $-3 < 3x^2 < 3$, so this means that $-1 < x < 1$, and the radius of convergence is 1.

Submissions Closed

Compute the Taylor series centred around $x = 0$ of the function $f(x) = x\cos(x^2/3)$. What is its formula?

A $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+1}}{3^{2n+1}(2n)!}$

B $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+2}}{3^{2n}(2n)!}$

C $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+1}}{3(2n)!}$

D none of the above

47/47 answered Ask Again

Open Closed Responses Correct 88%

Radius of Convergence

Compute the radius of convergence of

$$\sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+2}}{3^{2n}(2n)!}$$

Takeaway

To compute the interval of convergence of a substituted series, use the original interval of convergence and transform it

This should remind you of integration by substitution, and changing the bounds

Submissions Closed

Multiple answers: Multiple answers are accepted for this question

We know that the Taylor series for the function $\ln(1-x)$ about $x=0$ converges for $-1<x<1$. What is the interval of convergence for the function $\ln(8-x)$?

A $-8<x<8$ because $\ln(8-x)=\ln(8(1-x/8))=\ln(8)+\ln(1-x/8)$

B $-8<x<-6$ because we have moved the function to the left by 7 units

C $-1<x<1$ because we have not transformed the function in a way that will change the interval of convergence

D none of the above is completely correct

46/46 answered — Ask Again

Open | Closed | Responses | Correct | 88%

Fill in the Blanks

If a ______ series for $f(x)$ at $x = a$ converges to f for $|x - a| < R$, then the series found by term-by-term differentiation is the Taylor series for ______, and converges on the interval ______.

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Let's revisit our friend,

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

To estimate $erf(x)$ for small x, we will write it as a Taylor series.
Q: Write down three steps to computing the Taylor series of $erf(x)$ around $x = 0$.

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- Write the Taylor series for e^x
- Plug in $x = -t^2$
- Integrate term-by-term

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Let's do it:

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

so

$$e^{-t^2} = \sum_{n=0}^{\infty} \frac{(-1)^n t^{2n}}{n!}$$

Q: What is the Taylor series for $erf(x)$?

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Q: What is the Taylor series for $erf(x)$?

$$erf(x) = \frac{2}{\sqrt{\pi}} \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)(n!)}$$

Computing Series Using Taylor Polynomials

WolframAlpha says:

$$erf(1) = 0.842\cdots$$

We can use this to compute:

$$0.746 \approx erf(1)\frac{\sqrt{\pi}}{2} = \sum_{n=0}^{\infty} \frac{(-1)^n 1^{2n+1}}{(2n+1)(n!)}$$
$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)(n!)}$$

Submissions Closed

Find the exact sum of the series

$$\sum_{n=1}^{\infty} n(0.2)^{n-1}$$

44/44 answered

Ask Again

Open | Closed | Responses | Correct

88%

Takeaway

When we have a series, we can plug in a variable, x, then interpret it as a derivative or an integral of series that we know

Plans for the Future

For next time:
Watch the week 12 videos, and review section 10.3