Welcome to MAT136 LEC0501 (Assaf)

No more in-class TopHats. The software isn't working and I'm tired of fighting it.

S9.5 – Power Series & Convergence Interval

Assaf Bar-Natan

> “You and me got staying power yeah
> You and me we got staying power
> Staying power (I got it I got it)”
>
> –“Staying Power”, Queen

March 23, 2020

Using the Ratio Test on a Power Series

We are given the power series:

$$\sum_{n=1}^{\infty} C_n (x-a)^n$$

To check for convergence, apply the **ratio test**:

$$\lim_{n\to\infty} \left|\frac{C_{n+1}(x-a)^{n+1}}{C_n(x-a)^n}\right| = \lim_{n\to\infty} |x-a| \left|\frac{C_{n+1}}{C_n}\right| = |x-a| \lim_{n\to\infty} \left|\frac{C_{n+1}}{C_n}\right|$$

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The series $\sum C_n(x-a)^n$ converges when the above is less than 1.
Q: If $\lim_{n\to\infty}|C_{n+1}/C_n| = 3$, what is the radius of convergence?
A: We want $3|x-a| < 1$, so $|x-a| < \frac{1}{3}$, and this is the radius of convergence.

Variables, Indices, and Parameters

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We compute:

$$\lim_{n\to\infty} \frac{c^{(n+1)/2}}{c^{n/2}} \frac{n}{n+1} = \lim_{n\to\infty} \frac{n}{n+1} c^{1/2} = \sqrt{c}$$

So the radius of convergence is $\frac{1}{\sqrt{c}}$.

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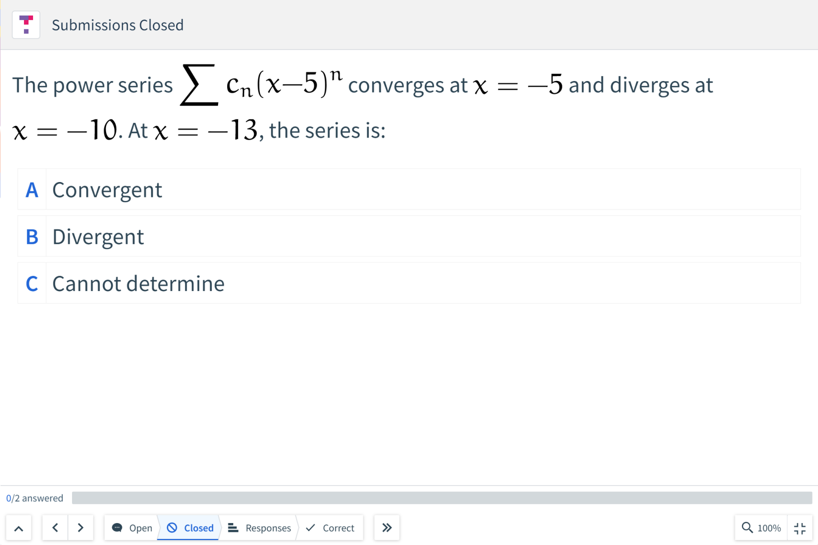
What is the interval of convergence of this power series?

The power series is centered at $x = a$, so it will converge for

$$a - \frac{1}{\sqrt{c}} < x < a + \frac{1}{\sqrt{c}}$$

Takeaway

In general, for $\sum c_n(x-a)^n$, the interval of convergence is centered at a.

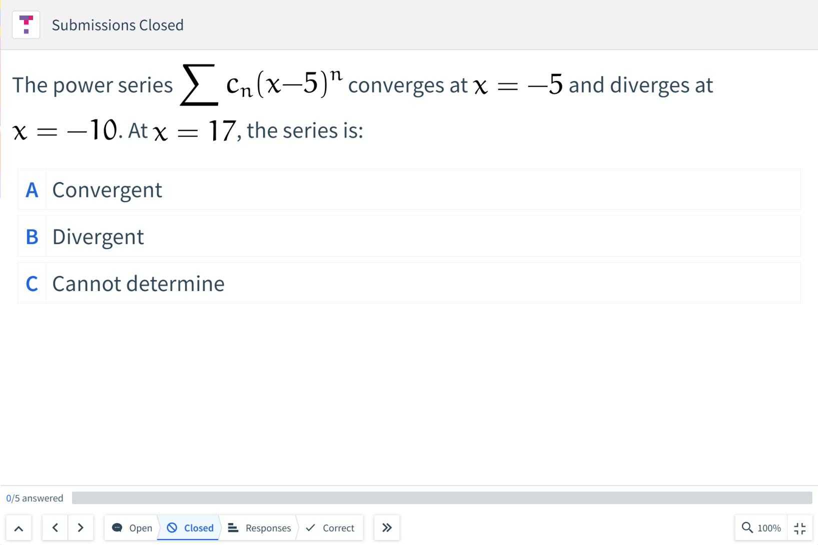


The power series $\sum c_n(x-5)^n$ converges at $x=-5$ and diverges at $x=-10$. At $x=-13$, the series is:

A Convergent

B Divergent

C Cannot determine

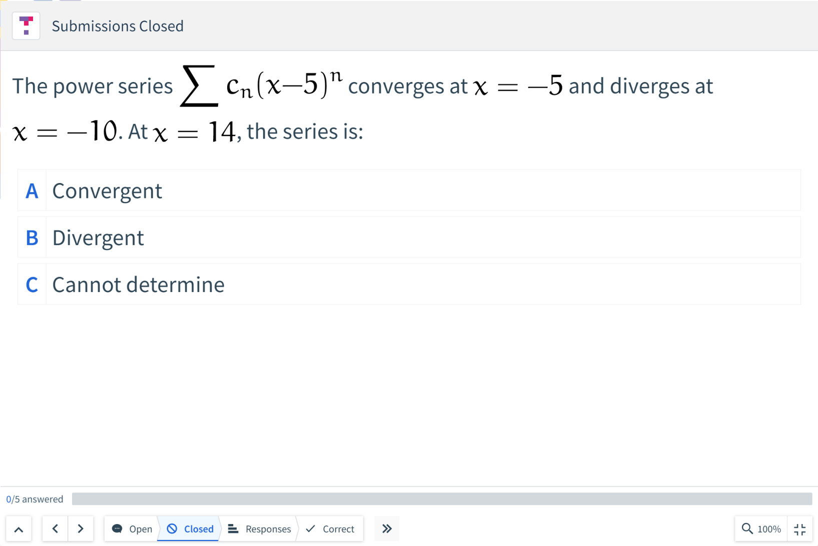


Submissions Closed

The power series $\sum c_n(x-5)^n$ converges at $x=-5$ and diverges at $x=-10$. At $x=17$, the series is:

- **A** Convergent
- **B** Divergent
- **C** Cannot determine

0/5 answered



The power series $\sum c_n(x-5)^n$ converges at $x=-5$ and diverges at $x=-10$. At $x=14$, the series is:

- **A** Convergent
- **B** Divergent
- **C** Cannot determine

What Possible Interval?

Draw a possible interval of convergence for $\sum c_n(x-5)^n$, given that the series converges at $x=-5$ and diverges at $x=-10$.

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Draw a possible interval of convergence for $\sum c_n(x-5)^n$, given that the series converges at $x=-5$ and diverges at $x=-10$.

We know that the interval needs to be centered at 5. Since the series converges at -5, this means that the radius of convergence is at least 10. Since the series diverges at $x=-10$, this means that the radius of convergence is less than 15. A possible interval of convergence is:

$$|x-5| < 11$$
$$-6 < x < 16$$

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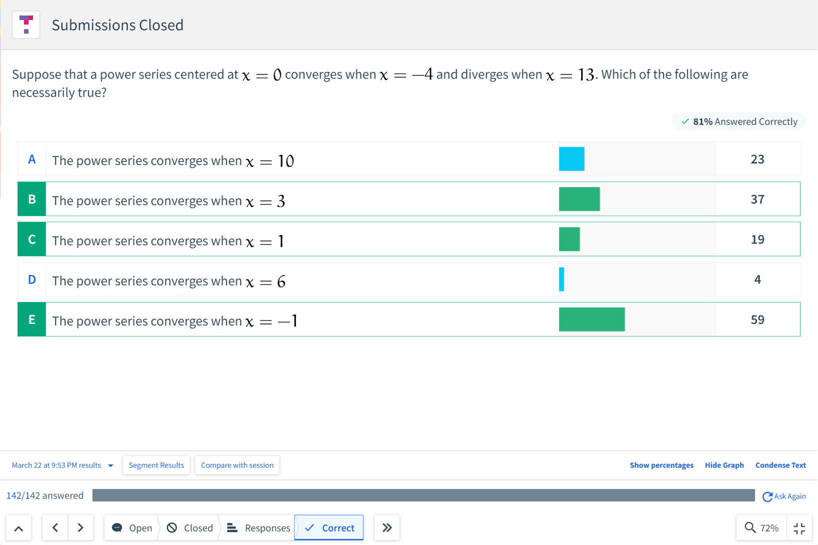
$$|x-5| < 11$$
$$-6 < x < 16$$

Note that the interval $|x-5| < 14$ (ie $-9 < x < 19$) is also possible

Plans for the Future

For next time:

Watch the week 11 videos, do WeBWork 10.2, actively read section 10.2



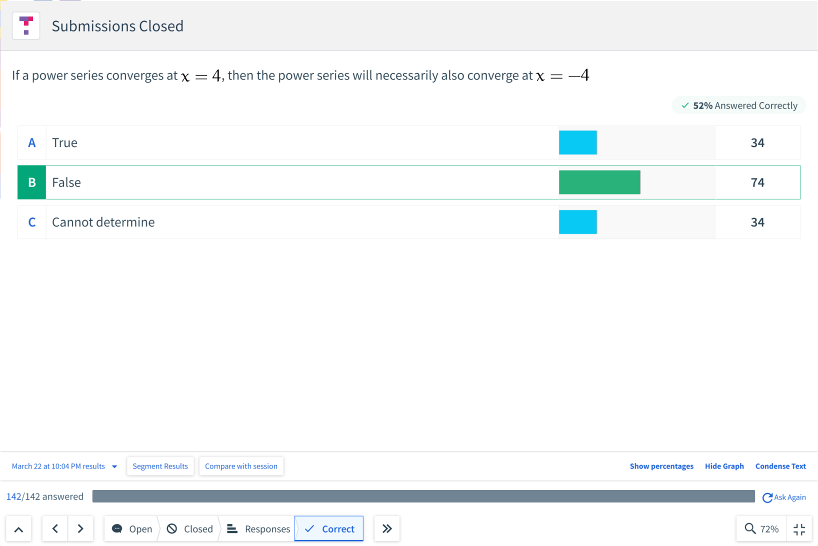
Submissions Closed

Suppose that a power series centered at $x = 0$ converges when $x = -4$ and diverges when $x = 13$. Which of the following are necessarily true?

✓ **81%** Answered Correctly

A	The power series converges when $x = 10$	23
B	The power series converges when $x = 3$	37
C	The power series converges when $x = 1$	19
D	The power series converges when $x = 6$	4
E	The power series converges when $x = -1$	59

142/142 answered



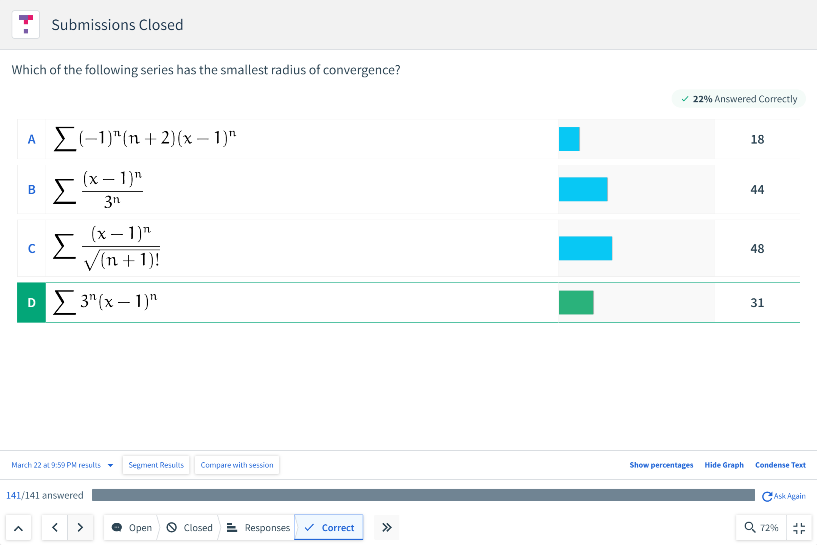
Submissions Closed

If a power series converges at $x = 4$, then the power series will necessarily also converge at $x = -4$

✓ **52%** Answered Correctly

A	True	34
B	False	74
C	Cannot determine	34

142/142 answered



Submissions Closed

Which of the following series has the smallest radius of convergence?

✓ **22%** Answered Correctly

A	$\sum (-1)^n (n+2)(x-1)^n$	18
B	$\sum \frac{(x-1)^n}{3^n}$	44
C	$\sum \frac{(x-1)^n}{\sqrt{(n+1)!}}$	48
D	$\sum 3^n (x-1)^n$	31

141/141 answered