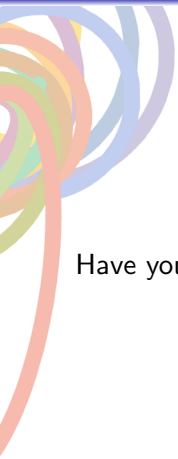


# Welcome to MAT135 LEC0501 (Assaf)



Have you formed a study group yet?



## S5.3 – The FUNdamental Theorem

Assaf Bar-Natan

“ F is for friends who do stuff together  
U is for you and me  
N is for anywhere and anytime at all  
Down here in the deep blue sea ”

–“ F.U.N Song ”, Spongebob

Jan. 10, 2020

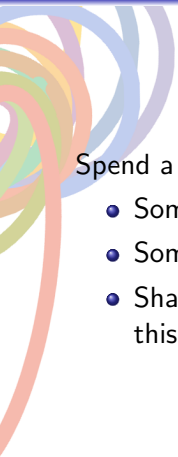
# Ice-Cream Sandwich



Spend a minute to think about:

- Something in the chapter that you've mastered.
- Something in the chapter that was hard.

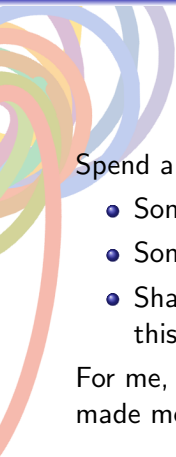
# Ice-Cream Sandwich



Spend a minute to think about:

- Something in the chapter that you've mastered.
- Something in the chapter that was hard.
- Share with your group what made something click for you in this chapter.

# Ice-Cream Sandwich

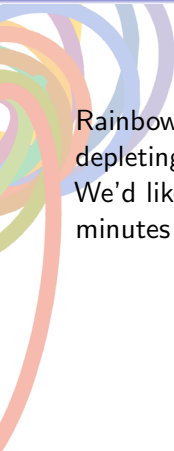


Spend a minute to think about:

- Something in the chapter that you've mastered.
- Something in the chapter that was hard.
- Share with your group what made something click for you in this chapter.

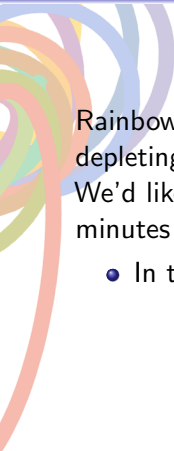
For me, the intuition for the F.T.C was something new that really made me understand what's going on.

# Intuition for the F.T.C



Rainbow, Marzipan, Blackie, and Lexi are eating from the cat-dish, depleting the Christmas left-overs at a rate of  $r(t)$  liters per minute. We'd like to find out how much the cats have eaten in the first five minutes of their feast.

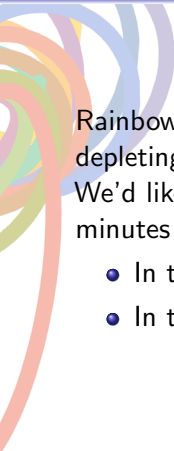
# Intuition for the F.T.C



Rainbow, Marzipan, Blackie, and Lexi are eating from the cat-dish, depleting the Christmas left-overs at a rate of  $r(t)$  liters per minute. We'd like to find out how much the cats have eaten in the first five minutes of their feast.

- In the 30 seconds, they eat approximately \_\_\_\_\_ liters.

# Intuition for the F.T.C

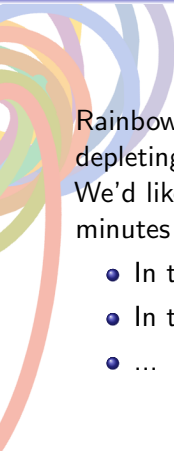


Rainbow, Marzipan, Blackie, and Lexi are eating from the cat-dish, depleting the Christmas left-overs at a rate of  $r(t)$  liters per minute. We'd like to find out how much the cats have eaten in the first five minutes of their feast.

- In the 30 seconds, they eat approximately \_\_\_\_\_ liters.
- In the next 30 seconds, they eat approximately \_\_\_\_\_ liters.



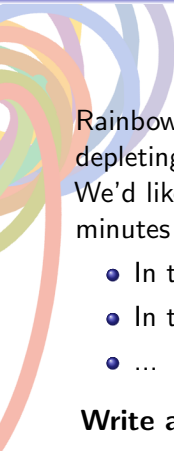
# Intuition for the F.T.C



Rainbow, Marzipan, Blackie, and Lexi are eating from the cat-dish, depleting the Christmas left-overs at a rate of  $r(t)$  liters per minute. We'd like to find out how much the cats have eaten in the first five minutes of their feast.

- In the 30 seconds, they eat approximately \_\_\_\_\_ liters.
- In the next 30 seconds, they eat approximately \_\_\_\_\_ liters.
- ...

# Intuition for the F.T.C

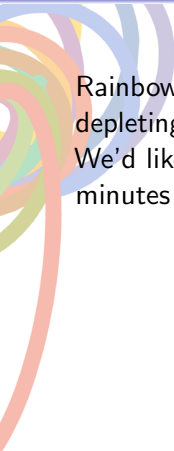


Rainbow, Marzipan, Blackie, and Lexi are eating from the cat-dish, depleting the Christmas left-overs at a rate of  $r(t)$  liters per minute. We'd like to find out how much the cats have eaten in the first five minutes of their feast.

- In the 30 seconds, they eat approximately \_\_\_\_\_ liters.
- In the next 30 seconds, they eat approximately \_\_\_\_\_ liters.
- ...

**Write an expression for the approximate amount of food the cats ate in five minutes. Use summation notation.**

# Intuition for the F.T.C



Rainbow, Marzipan, Blackie, and Lexi are eating from the cat-dish, depleting the Christmas left-overs at a rate of  $r(t)$  liters per minute. We'd like to find out how much the cats have eaten in the first five minutes of their feast.

They ate approximately:

# Intuition for the F.T.C

Rainbow, Marzipan, Blackie, and Lexi are eating from the cat-dish, depleting the Christmas left-overs at a rate of  $r(t)$  liters per minute. We'd like to find out how much the cats have eaten in the first five minutes of their feast.

They ate approximately:

$$\sum_{i=0}^9 r\left(\frac{i}{2}\right) \cdot \frac{1}{2}$$

**This looks like a Riemann sum!**

# Intuition for the F.T.C

Rainbow, Marzipan, Blackie, and Lexi are eating from the cat-dish, depleting the Christmas left-overs at a rate of  $r(t)$  liters per minute. We'd like to find out how much the cats have eaten in the first five minutes of their feast.

They ate approximately:

$$\sum_{i=0}^9 r\left(\frac{i}{2}\right) \cdot \frac{1}{2}$$

**This looks like a Riemann sum!**

**Write an expression for the exact amount of food the cats ate in five minutes.**

# Takeaway

**If  $f$  is a differentiable function on an interval  $[a, b]$  then**

$$\int_a^b f'(x)dx = f(b) - f(a).$$



Submissions Closed

Let  $f(x) = \log(\log(x))$ , where  $\log$  is taken with base  $e$ . Then the integral  $\int_3^5 f''(x) dx$  is (submit 0 if you don't have any idea how to do this)

✓ 36% Answered Correctly

-0.189 to -0.169	<div style="width: 36%; background-color: green;"></div>	68
1.991 to 2.011	<div style="width: 0%;"></div>	2
-5.009 to -4.989	<div style="width: 0%;"></div>	1
2.171 to 2.191	<div style="width: 0%;"></div>	1
-2.409 to -2.389	<div style="width: 0%;"></div>	1

January 9 at 7:12 PM results ▾

[Show percentages](#) [Hide Graph](#) [Condense Text](#)

188/188 answered

[Ask Again](#)

⏪ ⏩ 🔍 Open 🔒 Closed 📄 Responses ✓ Correct ⏭

🔍 100% 🏠

## Estimating using F.T.C

Rainbow, Marzipan, Blackie, and Lexi are eating from the cat-dish, depleting the Christmas left-overs at a rate of  $r(t)$  liters per minute. This quantity is measured in the table below:

$t$	0	2	3	4	5
$r(t)$	0.5	0.3	0.2	0.1	0.05

Give your best upper **or** lower estimate for the total amount of food the cats ate in the first five minutes.



# Estimating using F.T.C

Rainbow, Marzipan, Blackie, and Lexi are eating from the cat-dish, depleting the Christmas left-overs at a rate of  $r(t)$  liters per minute. This quantity is measured in the table below:

$t$	0	2	3	4	5
$r(t)$	0.5	0.3	0.2	0.1	0.05

Give your best upper **or** lower estimate for the total amount of food the cats ate in the first five minutes.

Find a group around you that estimated differently than you (ie, if you did a lower estimate, find a group who did an upper estimate), and explain to each other how you arrived at your estimates.

# Takeaway



**The fundamental theorem gives us a link between areas and rates!**



Submissions Closed

A bakery orders a special European butter especially for their cranberry-orange-pecan cookies. Let  $C(b)$  be the bakery's cost, in dollars, to buy  $b$  pounds of this special butter. It costs the bakery exactly \$3.50 less to buy 14 pounds butter than it does to buy 15 pounds of butter. Which of the following expressions represents this statement?

✓ 71% Answered Correctly

A	$\int_{14}^{15} C'(b) db = 3.5$		138
B	$\int_{14}^{15} C'(b) db = -3.5$		29
C	$C'(15) = 3.5$		20
D	$C'(15) = -3.5$		8

Invalid date Segment Results Compare with session

Show percentages Hide Graph Condense Text

195/195 answered

Ask Again

Open Closed Responses Correct

88%

 Submissions Closed

A bakery orders a special European butter especially for their cranberry-orange-pecan cookies. Let  $C(b)$  be the bakery's cost, in dollars, to buy  $b$  pounds of this special butter. Let  $K(b)$  be the amount of cookie dough, in cups, the bakery makes from  $b$  pounds of butter. If the bakery spends \$10 on butter, then it can make 20 cups of cookie dough. Which of the following expressions represents the statement?

✓ 90% Answered Correctly









A	$K(C(20)) = 10$	<input type="checkbox"/>	2
B	$C(K^{-1}(20)) = 10$	<input type="checkbox"/>	17
C	$C^{-1}(K(10)) = 20$	<input type="checkbox"/>	17
D	$K(C^{-1}(10)) = 20$	<input checked="" type="checkbox"/>	160
E	I've got no idea.	<input type="checkbox"/>	0

Invalid date  Segment Results Compare with session

Show percentages Hide Graph Condense Text

196/196 answered

 Ask Again





    Open  Closed  Responses  Correct 

 88% 

 Submissions Closed

A bakery orders a special European butter especially for their cranberry-orange-pecan cookies. Let  $K(b)$  be the amount of cookie dough, in cups, the bakery makes from  $b$  pounds of butter. 10 pounds of butter makes 40 cups more cookie dough than 5 pounds of butter. Which of the following expressions most accurately represents the statement?

✓ 87% Answered Correctly








A	$\int_5^{10} K(b) db = 40$		19
B	$\int_5^{10} K'(b) db = 40$		166
C	$K'(5) = 40$		3
D	$K'(10) = -40$		0
E	I've got no idea.		2

Invalid date [Segment Results](#) [Compare with session](#)

[Show percentages](#) [Hide Graph](#) [Condense Text](#)

190/190 answered

[Ask Again](#)

      [Correct](#) 

 88% 



Submissions Closed

A bakery orders a special European butter especially for their cranberry-orange-pecan cookies.

Let  $C(b)$  be the bakery's cost, in dollars, to buy  $b$  pounds of this special butter.

Let  $K(b)$  be the amount of cookie dough, in cups, the bakery makes from  $b$  pounds of butter

What are the units of  $\int_a^b K(C^{-1}(x)) dx$ ?

✓ 33% Answered Correctly

A	cups-pounds	<div style="width: 20%;"></div>	66
B	dollars-cups/pound	<div style="width: 10%;"></div>	32
C	dollar-cups	<div style="width: 20%;"></div>	63
D	dollar-pounds/cup	<div style="width: 10%;"></div>	25
E	I've got no idea.	<div style="width: 2%;"></div>	6

Invalid date ▾ Segment Results Compare with session

Show percentages Hide Graph Condense Text


192/192 answered

Ask Again

⏪ ⏩ 🔍 Open 🔒 Closed 📄 Responses ✓ Correct ⏪

🔍 88% 🏠

# Takeaway



**When doing interpretation questions, work slowly, and watch for units!**

# Plans for the Future



For next time:

**WeBWork 5.4 and read section 5.4**