# Welcome to MAT136 LEC0501 (Assaf)

Online final exam is happening April 9 between 1:30pm and 5:30pm EST. See main course page for details (none posted yet).

# S9.3 – Series & The Ratio Test

Assaf Bar-Natan

"Life is a series of hellos and goodbyes
I'm afraid it's time for goodbye again
Say goodbye to Hollywood
Say goodbye my baby"

–"Say Goodbye to Hollywood", Billy Joel

March 20, 2020

# Fill in the Blanks

We have a series, $\sum a_n$.

If the ratios $\frac{a_{n+1}}{a_n}$ approach $L$, and $L < 1$, then the series $\sum a_n$ grows ______ a geometric series with factor ______, which is ___ $(<, >, =)$ 1. Hence, the series ______.

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Submissions Closed

Suppose that

$$\lim_{k\to\infty} \frac{|a_{k+1}|}{|a_k|} = 1.$$

Then the series $\sum a_k$ neither converges nor diverges.

✓ **58%** Answered Correctly

| | | |
|---|---|---|
| A | True | 26 |
| B | False | 36 |

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62/62 answered Ask Again

Open Closed Responses ✓ Correct 72%

# Why the Ratio Test

Let's assume that for any sufficiently large $n$, $\frac{a_{n+1}}{a_n} \approx L$. Then:

$$a_{k+1} \approx La_k$$
$$a_{k+2} \approx La_{k+1} \approx L^2 a_k$$

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Continuing in this manner, we get:

$$a_k + a_{k+1} + a_{k+2} + \cdots \approx a_k \left(1 + L + L^2 + L^3 + \cdots\right)$$

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If $L < 1$, then the right hand side is a geometric series, which converges!
If $L = 0$, replace all $\approx$ with $<$, and replace $L$ with $\frac{1}{2}$

# Takeaway

**The ratio test measures how much a series looks like a geometric series. If the limit of the ratio $\frac{a_{n+1}}{a_n}$ is $< 1$, the series converges, and if it is $> 1$, it diverges. Just like a geometric series!**

# Obie and Limits

Obie (the bully cat) says:

"In examining the series:

$$\sum_{n=0}^{\infty} \frac{n}{(1.05)^n} = 0.95 + 1.181 + 2.59 + 3.29 + \cdots$$

I notice that the terms are getting larger, so $L > 1$. Thus, by the ratio test, this series diverges."

Is Obie correct?

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Is Obie correct?
If this still confuses you, write a star in your notebook to go over this later

# Takeaway

**We really do need to take a limit in the ratio test. We don't care what the series' terms do early. The limit captures this "end behaviour"**

Submissions Closed

For the series $\sum_{k=1}^{\infty} \frac{k+1}{k!}$, which test would you use?

✓ **77%** Answered Correctly

| | | |
|---|---|---|
| A | Ratio Test | 47 |
| B | Integral Test | 8 |
| C | Divergence Test | 6 |

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61/62 answered | Ask Again

Open | Closed | Responses | Correct | 72%

Submissions Closed

For the series $\sum_{k=1}^{\infty} \frac{k}{(k+1)^2}$, which test would you use?

✓ 51% Answered Correctly

| | | |
|---|---|---|
| A | Ratio Test | 14 |
| B | Integral Test | 26 |
| C | Divergence Test | 11 |

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51/51 answered · Ask Again

Open · Closed · Responses · ✓ Correct · 72%

Submissions Closed

For the series $\sum_{k=1}^{\infty} \frac{k}{k+1}$, which test would you use?

✓ **34%** Answered Correctly

| | | |
|---|---|---|
| A | Ratio Test | 27 |
| B | Integral Test | 12 |
| C | Divergence Test | 20 |

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59/60 answered · Ask Again

Open · Closed · Responses · ✓ Correct · 72%

# Inconclusive Test Results

Write a series which diverges, but for which the ratio test gives a limit of 1.

Challenge: write a series which converges, but for which the ratio test gives a limit of 1.

# Plans for the Future

For next time:

**Do WeBWork 9.5, actively read section 9.5, and watch the videos!**

Submissions Closed

Which test (or tests) can you use to determine if the following series converges?

$$\sum_{k=1}^{\infty} e^{-k}$$

✓ **67%** Answered Correctly

| | | |
|---|---|---|
| A | Divergence Test | 53 |
| B | Integral Test | 76 |
| C | Ratio Test | 30 |

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159/159 answered | Ask Again

Open | Closed | Responses | Correct | 72%

Submissions Closed

Which test (or tests) can you use to determine if the following series converges?

$$\sum_{k=1}^{\infty} e^k$$

✓ **100%** Answered Correctly

| | | |
|---|---|---|
| A | Divergence Test | 42 |
| B | Integral Test | 72 |
| C | Ratio Test | 45 |

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159/159 answered — Ask Again

Open | Closed | Responses | ✓ Correct | 72%

Submissions Closed

Suppose that you have a series that has negative terms as well as positive terms. Which of the following tests could you still try?

✓ 77% Answered Correctly

| | | |
|---|---|---|
| A | Divergence Test | 49 |
| B | Integral Test | 37 |
| C | Ratio Test | 72 |

Invalid date | Segment Results | Compare with session | Show percentages | Hide Graph | Condense Text

158/158 answered | Ask Again

Open | Closed | Responses | ✓ Correct | 72%