Welcome to MAT136 LEC0501 (Assaf)

Online final exam is happening April 9 between 1:30pm and 5:30pm EST. See main course page for details (none posted yet).

S9.3 – Series & The Ratio Test

Assaf Bar-Natan

"Life is a series of hellos and goodbyes I'm afraid it's time for goodbye again Say goodbye to Hollywood Say goodbye my baby"

- "Say Goodbye to Hollywood", Billy Joel

March 20, 2020

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We have a series, $\sum a_n$.

If the ratios $\frac{a_{n+1}}{a_n}$ approach *L*, and *L* < 1, then the series $\sum a_n$ grows ______ a geometric series with factor ______, which is _____ (<, >, =) 1. Hence, the series ______.

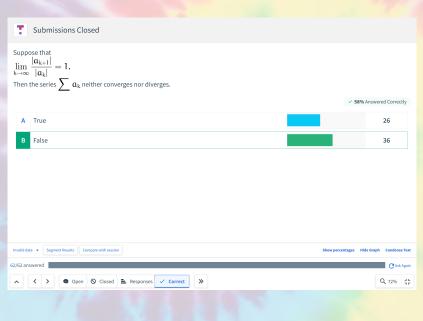
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If the ratios $\frac{a_{n+1}}{a_n}$ approach *L*, and *L* > 1, then the series $\sum a_n$ grows _____ a geometric series with factor _____, which is ____ (<, >, =) 1. Hence, the series _____.

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Let's assume that for any sufficiently large n, $\frac{a_{n+1}}{a_n} \approx L$. Then:

 $\begin{aligned} a_{k+1} &\approx La_k \\ a_{k+2} &\approx La_{k+1} &\approx L^2 a_k \end{aligned}$

Continuing in this manner, we get:

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Let's assume that for any sufficiently large n, $\frac{a_{n+1}}{a_n} \approx L$. Then:

 $\frac{a_{k+1}}{a_{k+2}} \approx La_k$ $a_{k+2} \approx La_{k+1} \approx L^2 a_k$

Continuing in this manner, we get:

$$\mathsf{a}_k + \mathsf{a}_{k+1} + \mathsf{a}_{k+2} + \cdots \approx \mathsf{a}_k \left(1 + \mathsf{L} + \mathsf{L}^2 + \mathsf{L}^3 + \cdots \right)$$

If L < 1, then the right hand side is a geometric series, which converges!

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Continuing in this manner, we get:

$$a_k + a_{k+1} + a_{k+2} + \cdots \approx a_k \left(1 + L + L^2 + L^3 + \cdots \right)$$

If L < 1, then the right hand side is a geometric series, which converges! If L = 0, replace all \approx with <, and replace L with $\frac{1}{2}$

The ratio test measures how much a series looks like a geometric series. If the limit of the ratio $\frac{a_{n+1}}{a_n}$ is < 1, the series converges, and if it is > 1, it diverges. Just like a geometric series!

Obie (the bully cat) says:

"In examining the series:

$$\sum_{n=0}^{\infty} \frac{n}{(1.05)^n} = 0.95 + 1.181 + 2.59 + 3.29 + \cdots$$

I notice that the terms are getting larger, so L > 1. Thus, by the ratio test, this series diverges."

Is Obie correct?

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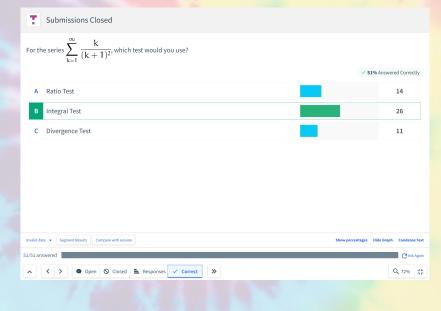
Is Obie correct? If this still confuses you, write a star in your notebook to go over this later

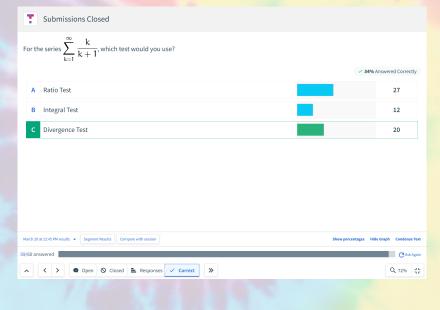
We really do need to take a limit in the ratio test. We don't care what the series' terms do early. The limit captures this "end behaviour"



77% Answered Correctly







Inconclusive Test Results

Write a series which diverges, but for which the ratio test gives a limit of 1. Challenge: write a series which converges, but for which the ratio test gives a limit of 1.

For next time: Do WeBWork 9.5, actively read section 9.5, and watch the videos!

\mathbf{T} Submissions Closed



✓ 67% Answered Correctly

Α	Divergence Test				
в	Integral Test	76			
с	Ratio Test	30			

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\mathbf{T}_{i} Submissions Closed

Which test (or tests) can you use to determine if the following series converges? $\sum_{k=1}^{\infty} e^k$

✓ 100% Answered Correctly

A Divergence Test	42
B Integral Test	72
C Ratio Test	45

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T Submissions Closed

Suppose that you have a series that has negative terms as well as positive terms. Which of the following tests could you still try?



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