Welcome to MAT136 LEC0501 (Assaf)

How similar are other online classes to this one? What's different?
Answer in the chat.

S9.3 – Series & Convergence

Assaf Bar-Natan

"One thing I can tell you is
You got to be free
Come together, right now
Over me"

–"Come Together", The Beatles

March 18, 2020

Fill in the Blanks

- We say that a series $\sum_{k=1}^{\infty} a_k$ c_____ if the p_____ s_____, $\sum_{k=1}^{n} a_k$ converge
- We define the value of a series as the _____ of the partial sums.
- The series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ _____ if $p \leq 1$, by the _____-test

Partial Sums and Convergence

When we write:

$$\sum_{k=1}^{\infty} a_k$$

what we really mean is:

$$\lim_{n \to \infty} \sum_{k=1}^{n} a_k$$

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If we write $S_n = \sum_{k=1}^{n} a_k$, and call it the **partial sum**, then the series $\sum_{k=1}^{\infty} a_k$ converges when $\lim_{n\to\infty} S_n$ converges.

Partial Sums – Geometric Series

Consider the series:

$$1 + 0.2 + (0.2)^2 + (0.2)^3 + \cdots$$

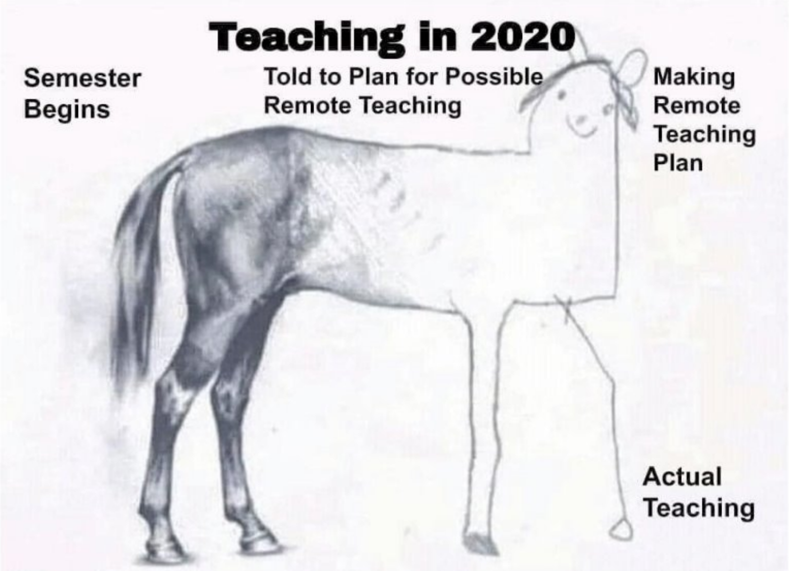
- What is a_k?
- What is S_n?
- What is $\lim_{n\to\infty} S_n$?
- What integral do we use in the integral test?

Partial Sums – Geometric Series

Consider the series:

$$1 + 0.2 + (0.2)^2 + (0.2)^3 + \cdots$$

- What is a_k? $a_k = (0.2)^k$
- What is S_n? $S_n = \frac{1-(0.2)^{n+1}}{0.8}$
- What is $\lim_{n\to\infty} S_n$? $\frac{1}{0.8}$
- What integral do we use in the integral test? **We use the integrand** $(0.2)^x$


Teaching in 2020
Semester Begins
Told to Plan for Possible Remote Teaching
Making Remote Teaching Plan
Actual Teaching

The Integral Idea

Suppose $a_n = f(n)$, where $f(x)$ is decreasing and positive.

- If $\int_1^\infty f(x)dx$ diverges, then $\sum a_n$ diverges.
- If $\int_1^\infty f(x)dx$ converges, then $\sum a_n$ converges.

Q: Does the series:

$$e^4 - 0.2 + \pi + 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \cdots$$

converge?

The Integral Idea

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Q: Does the series:

$$e^4 - 0.2 + \pi + 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \cdots$$

converge?

A: Yes! We only care about the tail of the series, which converges by the integral test.

Submissions Closed

The series $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$ converges

✓ **69%** Answered Correctly

A	True and I am confident in my answer.	13
B	True and I am not confident in my answer.	11
C	False and I am not confident in my answer.	25
D	False and I am confident in my answer.	29

Invalid date | Segment Results | Compare with session | **Show percentages** | **Hide Graph** | **Condense Text**

78/78 answered | Ask Again

Open | Closed | Responses | Correct | 72%

Lexi's Series

Lexi, the tail-less cat (she was born that way) is practicing her convergence properties. She writes:

" I want to see if the series $\sum(\frac{1}{n} - \frac{1}{n+1})$ converges. I'll split it up to get:

$$\sum_{n=1}^{\infty} \frac{1}{n} - \frac{1}{n+1} = \sum_{n=1}^{\infty} \frac{1}{n} - \sum_{n=1}^{\infty} \frac{1}{n+1}$$

The series on the right is the Harmonic series, which diverges, so the whole thing diverges."

Is Lexi's reasoning correct?

Submissions Closed

The series $\sum_{n=1}^{\infty} \frac{1}{n} - \frac{1}{n+1}$ converges

✓ **62%** Answered Correctly

A	True and I am confident in my answer.	17
B	True and I am not confident in my answer.	28
C	False and I am not confident in my answer.	20
D	False and I am confident in my answer.	8

March 18 at 12:10 PM results · Segment Results · Compare with session · Show percentages · Hide Graph · Condense Text

73/76 answered · Ask Again

Open · Closed · Responses · ✓ Correct · 72%

Takeaway

When all else fails, look at the partial sums!

Plans for the Future

For next time:

Do WeBWork 9.3 and actively read section 9.3

Submissions Closed

True / False: Since $\lim_{n\to\infty} 1/n = 0$, $\sum_{n=1}^{\infty} 1/n$ converges.

A	True, and I am very certain
B	True, but I am not very certain
C	False, but I am not very certain
D	False, and I am very certain

132/132 answered Ask Again

Open Closed Responses Correct 88%

Submissions Closed

True / False: Since $\lim_{n\to\infty} 1/n = 0$, $\sum_{n=1}^{\infty} 1/n$ converges.

✓ **20%** Answered Correctly

A	True, and I am very certain	41
B	True, but I am not very certain	47
C	False, but I am not very certain	18
D	False, and I am very certain	26

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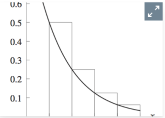
132/132 answered | Ask Again

Open | Closed | Responses | ✓ Correct | 88%

Submissions Closed

The graph below is of $y = 2^{-x}$ and the area of the sequential rectangles is $1/2, 1/4, 1/8, 1/16, \ldots$. Since we know that $\int_1^\infty 2^{-x}\,dx$ converges, what can you conclude directly from this picture?



✓ 18% Answered Correctly

A	The series $\sum_{k=1}^{\infty} 2^{-n}$ converges	81
B	The series $\sum_{k=1}^{\infty} 2^{-n}$ diverges	27
C	We cannot get any information about the series $\sum_{k=1}^{\infty} 2^{-n}$ directly from this picture	23

Invalid date | Segment Results | Compare with session | Show percentages | Hide Graph | Condense Text

131/131 answered | Ask Again

Open | Closed | Responses | Correct | 88%

Submissions Closed

$$\sum_{n=1}^{\infty}(1+(-1)^n)....$$

✓ 59% Answered Correctly

A	converges	27
B	diverges	77
C	we cannot determine with what we've learned so far	26

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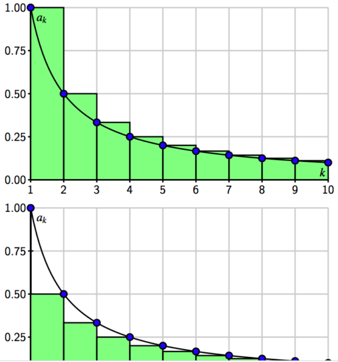
130/130 answered · Ask Again

Open · Closed · Responses · ✓ Correct · 72%

Submissions Closed

Let $f(x)$ be a function such that $f(n) = a_n$. Suppose that $\int_1 f(x)\,dx$ diverges. Which picture below implies that $\sum_{k=1} a_k$ also diverges?



131/131 answered

Ask Again

Open | Closed | Responses | Correct

72%