Welcome to MAT136 LEC0501 (Assaf)

`https://www.youtube.com/watch?v=Kas0tIxDvrg`

An interesting video on COVID-19 modeling and exponential growth.

Q: What model (SI, SIR, or SIS) is this video using?

S8.4 – Density and Slicing

Assaf Bar-Natan

" Come gather 'round people
Wherever you roam
And admit that the waters
Around you have grown"

–"The Times They Are 'a Changin'", Simon and Garfunkel

March 9, 2020

WeBWork Round Robin

In your groups, go in a circle, and:

- Say a problem from the WeBWork you struggled with.

WeBWork Round Robin

In your groups, go in a circle, and:

- Say a problem from the WeBWork you struggled with.
- Discuss the solution to each problem that the group mentioned.

WeBWork Round Robin

In your groups, go in a circle, and:

- Say a problem from the WeBWork you struggled with.
- Discuss the solution to each problem that the group mentioned.
- Write a hint for a student struggling with the problem.

Takeaway

In life, and on the exam, you will be asked to communicate your math using complete sentences.

The writing exercises we do in class are for your practice!

What is Density?

Flood has a long tail, and the fur-density is given by a function, $h(l)\frac{\text{hairs}}{\text{m}}$, where l is the length along her tail. If Flood's tail is 30cm long, how many hairs does Flood have?

What is Density?

Flood has a long tail, and the fur-density is given by a function, $h(l)\frac{\text{hairs}}{\text{m}}$, where l is the length along her tail. If Flood's tail is 30cm long, how many hairs does Flood have?

$$\text{Hairs} \approx \sum h(l)\Delta l = \int_a^b h(l)dl$$

Q: What are a and b? (Hint: units!)

What is Density?

Flood has a long tail, and the fur-density is given by a function, $h(l)\frac{\text{hairs}}{\text{m}}$, where l is the length along her tail. If Flood's tail is 30cm long, how many hairs does Flood have?

$$\text{Hairs} \approx \sum h(l)\Delta l = \int_a^b h(l)dl$$

Q: What are a and b? (Hint: units!) $a = 0$ and $b = 0.3\text{m} = 30\text{cm}$.

Takeaway

Always make sure that the units work out!

Torontopolis

The fictional city of Torontopolis radially has a population density of $4000e^{-0.02r^2}$ people per km^2, where r is the radius (in km) from the CM-tower.

We are interested in finding the total population living within a certain radius of the CM-tower.

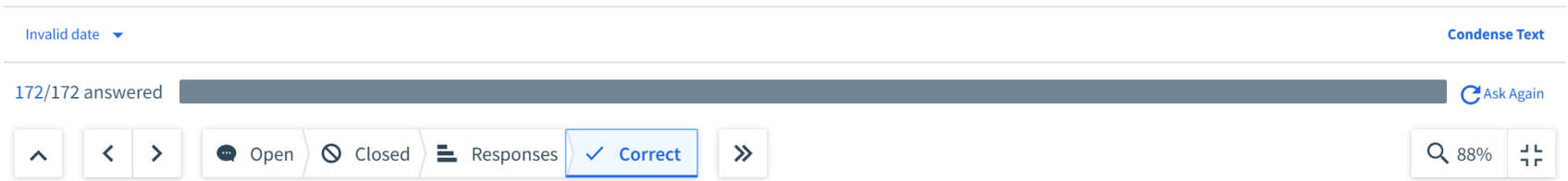
Submissions Closed

Put the steps for solving a slicing problem in order.

✓ **59%** Answered Correctly

Correct Order

B	Slice the object or process into pieces where you can approximate quantity.
E	Approximate the quantity on each slice.
F	Add up the slices to get an approximation for the total.
A	Take a limit as the number of slices approaches infinity to get the exact value for the total.
D	Interpret your limit as an integral.
C	Use the FTC to find an exact value for the total.



Slice object where density is constant

Discussion: Along what "slices" of Torontopolis is the population density approximately constant?

Slice object where density is constant

Discussion: Along what "slices" of Torontopolis is the population density approximately constant?

A: Annuli of small thickness centered at the CM-tower.

Submissions Closed

True or False: A different city, Montrealville, occupies a region in the xy-plane, with population density $\delta(y) = 1 + y$. To set up an integral representing the total population in the city, we should slice the region into...

✓ **55%** Answered Correctly

A	Pieces that run parallel to the x axis	96
B	Annuli around a center point	16
C	Pieces that run parallel to the y axis	54
D	Depends on the shape of Montrealville	7

Invalid date | Segment Results | Compare with session | Show percentages | Hide Graph | Condense Text

173/173 answered | Ask Again

Open | Closed | Responses | Correct | 88%

Add up slices

Discussion: What is the total population living on an annulus of radius r_i and of width Δr?

Add up slices

Discussion: What is the total population living on an annulus of radius r_i and of width Δr?

A: $4000e^{-0.02r^2} \times 2\pi r \times \Delta r$

Interpret as Riemann sum

Discussion: What is the total number of people who live within a $3km$ radius of the CM-tower? Write your answer as a Riemann sum.

Interpret as Riemann sum

Discussion: What is the total number of people who live within a $3km$ radius of the CM-tower? Write your answer as a Riemann sum.

A: We partition the interval $[0, 3]$ into n pieces. So $\Delta r = \frac{3}{n}$.

What is r_i?

Interpret as Riemann sum

Discussion: What is the total number of people who live within a $3km$ radius of the CM-tower? Write your answer as a Riemann sum.

A: We partition the interval $[0,3]$ into n pieces. So $\Delta r = \frac{3}{n}$.

What is r_i?

A: $r_i = \frac{3i}{n}$, so the sum becomes:

Interpret as Riemann sum

Discussion: What is the total number of people who live within a $3km$ radius of the CM-tower? Write your answer as a Riemann sum.

A: We partition the interval $[0,3]$ into n pieces. So $\Delta r = \frac{3}{n}$.

What is r_i?

A: $r_i = \frac{3i}{n}$, so the sum becomes:

$$\sum_{i=1}^{n} 2\pi r_i \times 4000 e^{-0.02 r_i^2} \times \frac{3}{n}$$

To get the true quantity, take the limit.

Submissions Closed

We've seen that the number of people who live within 2km of the CM tower in Torontopolis is given by $\lim_{n\to\infty}\sum_{i=1}^{n} 8000\pi\frac{3i}{n}e^{-0.02\times(3i/n)^2}\frac{3}{n}$. What will evaluate this?

✓ **46%** Answered Correctly

A	$8000\pi\int_0^1 re^{-0.02r^2}dr$	9
B	$8000\pi\int_0^1 9re^{-0.02\times 9r^2}dr$	81
C	$8000\pi\int_0^1 3re^{-0.02\times 3r^2}dr$	12
D	$8000\pi\int_0^1 3re^{-0.02\times 9r^2}dr$	69

March 8 at 10:14 PM results | Segment Results | Compare with session | Show percentages | Hide Graph | Condense Text

176/176 answered | Ask Again

Open | Closed | Responses | Correct | 88%

Compute the Integral

The total number of people who live within a 3km radius of the CM-tower is:

$$8000\pi \int_0^1 9re^{-0.02\times 9r^2}\,dr = 8000\pi \int_0^3 re^{-0.02r^2}\,dr$$

Compute the Integral

The total number of people who live within a 3km radius of the CM-tower is:

$$8000\pi \int_0^1 9re^{-0.02\times 9r^2}\,dr = 8000\pi \int_0^3 re^{-0.02r^2}\,dr \approx 103,000$$

Takeaway

Reminder: for ALL slicing problems, you need to show all the steps on the exam!

Plans for the Future

For next time:

Go over WeBWork 8.4 and section 8.4

Ban cars on campus