Welcome to MAT136 LEC0501 (Assaf)

We continue to solve $\int \sqrt{\tan(x)}$. After computing the last integral, and subbing in everything...

$$\begin{aligned}\int \sqrt{\tan(x)}dx = &\frac{1}{2\sqrt{2}}\log(\tan(x) - \sqrt{2\tan(x)} + 1)\\ &- \frac{1}{2\sqrt{2}}\log(\tan(x) + \sqrt{2\tan(x)} + 1)\\ &+ \frac{1}{\sqrt{2}}\tan^{-1}(\sqrt{2\tan(x)} + 1)\\ &- \frac{1}{\sqrt{2}}\tan^{-1}(1 - \sqrt{2\tan(x)})\end{aligned}$$

Easy, right?

Areas and Volumes – Rotating, Spinning, and a Bit of Slicing

Assaf Bar-Natan

"My head is in a spin,
My feet don't touch the ground.
Because you're near to me
My head goes round and round."

–"Feels Like I'm in Love", Kelly Marie

March 6, 2020

CIQ summary

- You were most engaged when using TopHat. Specifically, discussing with groups, and discussing things together.
- You were engaged during the lectures about COVID-19, SIR model, and the Excel spreadsheet activity
- You were sad that people leave before class is over, or talk over me.
- Some of you were distanced when doing the SIR model stuff. A lot of you were confused at slope fields.
- At times, the lecture was moving fast, and you disliked skipped questions.
- You did not gain a lot from classes when you did not do the reading.

CIQ summary – Cont.

Things you liked:

- Taking up WeBWork and TopHat questions (through discussions, or on slides)
- The amount of interaction and group discussion
- Traditional lecturing
- Each other

Things that confused you:

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- idk (this annoyed a lot of of you, too)

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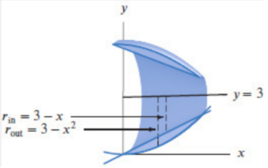
- Each other

Things that confused you:

- idk (this annoyed a lot of of you, too)
- Going too fast

No Correct Answer

Draw a cross-section of this shape, when sliced with vertical sections (ie, planes perpendicular to the x-axis). What are the dimensions of these cross-sections?



Reply

Ordered by Last Name

Thomas Aalbers

a day ago

Comments 0 1

Mohamed Ali

a day ago

153/153 answered

Resume

100%

CIQ summary – Cont.

Things that surprised you:

CIQ summary – Cont.

Things that surprised you:

- 0/1000000000

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- Difference in marks between those who attended the class on the day of the mid term and those who did not

CIQ summary – Cont.

Things that surprised you:

- 0/1000000000
- "The friends I made in this class"
- SIR/Coronavirus modeling
- Difference in marks between those who attended the class on the day of the mid term and those who did not
- Cats?

A First Application of Slices

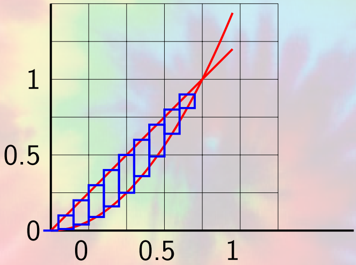
To find the arc-length of a function:

$$\Delta \text{Arc length} = \sqrt{\Delta x^2 + \Delta y^2}$$
$$= \sqrt{\Delta x^2 + (f'(x)^2)\Delta x^2}$$
$$= \sqrt{1 + (f')^2}\Delta x$$

Integrate to get $Arclength = \int \sqrt{1 + (f')^2} dx$

Slices – Areas and Volumes

Previously, on MAT136:



Taking the slices to be really small...

$$\text{Area} = \int_a^b (f(x) - g(x))dx$$

Now, on MAT136: **Find the volume of the solid obtained by rotating the region in the first quadrant bounded by and $y = x^2$, $y = x$ around the $y = 3$ axis.**

Q: What does the base region look like?

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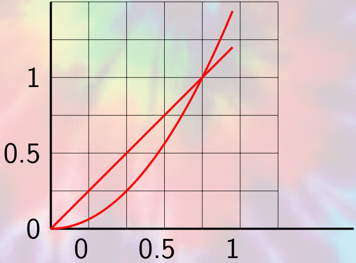
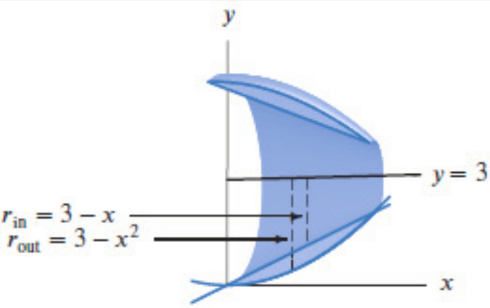
Q: What does the base region look like?



Slices – Areas and Volumes

rotate the region between the curves around the line $y = 3$



Use slices here, and make them really small...

Finding the Volume

Get into groups.

- In your groups, draw the cross-section of the slices

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  $V = \Delta x \pi((3 - x^2)^2 - (3 - x)^2)$
- Write an integral that computes the total volume

Finding the Volume

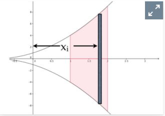
Get into groups.

- In your groups, draw the cross-section of the slices
- Write an expression for the volume of each slice.
  $V = \Delta x \pi((3 - x^2)^2 - (3 - x)^2)$
- Write an integral that computes the total volume

$$\int_0^1 \pi((3 - x^2)^2 - (3 - x)^2)dx$$

Submissions Closed

We rotate the graph of $y = (x+1)^2$ around the x-axis. The approximate volume of the slice of the solid that is x_i units away from the y-axis is given by:



✓ 73% Answered Correctly

A	$(x_i + 1)^2 \Delta x$	3
B	$\pi x_i^2 \Delta x$	11
C	$\pi (x_i + 1)^2 \Delta x$	30
D	$\pi (x_i + 1)^4 \Delta x$	119

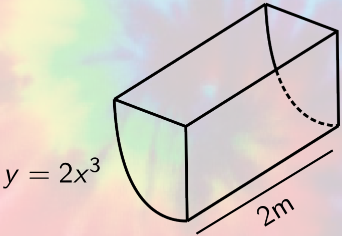
March 6 at 11:00 AM results · Segment Results · Compare with session · Show percentages · Hide Graph · Condense Text

163/163 answered · Ask Again

Open · Closed · Responses · Correct · 100%

Cats and Troughs

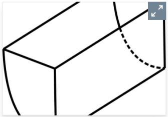
The cats are stealing food from the sheep's trough (pictured below schematically):



Assume that the trough has a cubic cross-section $y = 2x^3$, and that its length is $2m$. At the start of the day, the food in the trough is $0.4m$ high in the trough. At the end of the day, it's $0.39m$ high. How much food did the cats eat?

Submissions Closed

We can slice the volume of food at the start of the day using vertical slices or horizontal slices. Each of these slices then has a different area. Match the type of slicing to the formula giving the area of the slice.



✓ **20%** Answered Correctly

Correct Order

1	Horizontal slice at some y	→	B	$2 \times \sqrt[3]{y/2}$	48
2	Vertical slice at some x	→	A	$2 \times (0.4 - 2x^3)$	84

March 6 at 12:52 AM results

Condense Text

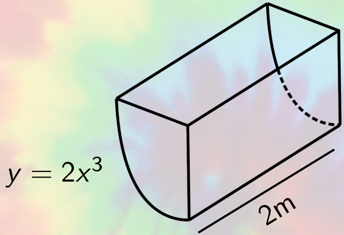
155/155 answered

Ask Again

Open Closed Responses Correct

100%

Cats and Troughs



Vertical Slices

$$\Delta\mathrm{V}(\text{start of day})$$
$$= 2 \times (0.4 - 2x^3) \times \Delta x$$

Horizontal Slices

$$\Delta\mathrm{V}(\text{start of day})$$
$$= 2 \times \sqrt[3]{y/2} \times \Delta y$$

Submissions Closed

We can slice the volume of food at the start of the day in two ways. Which of the following give an integral that evaluates the volume of food at the start of the day?

✓ 63% Answered Correctly

A	$2\int_0^{0.4} (0.4 - 2t^3)\,dt$	44
B	$2\int_0^{\sqrt[3]{0.2}} (0.4 - 2t^3)\,dt$	39
C	$2\int_0^{0.4} \sqrt[3]{t/2}\,dt$	56
D	$2\int_0^{\sqrt[3]{0.4/2}} \sqrt[3]{t/2}\,dt$	13

March 6 at 1:45 PM results | Segment Results | Compare with session | Show percentages | Hide Graph | Condense Text

152/152 answered | Ask Again

Open | Closed | Responses | Correct | 88%

Cats and Troughs

At the start of the day...

Vertical Slices

$$\Delta \mathrm{V} = 2 \times (0.4 - 2x^3) \times \Delta x$$

$$\mathrm{V} = 2 \int_0^{\sqrt[3]{0.2}} (0.4 - 2x^3)dx$$

Horizontal Slices

$$\Delta \mathrm{V} = 2 \times \sqrt[3]{y/2} \times \Delta y$$

$$\mathrm{V} = 2 \int_0^{0.4} \sqrt[3]{y/2}dy$$

Both are equal to ≈ 0.3508

Cats and Troughs

So how much did the cats eat?

Cats and Troughs

So how much did the cats eat?

$$\begin{aligned}
\text{V(start of day)} - \text{V(end of day)} \\
&= 2\int_0^{0.4} \sqrt[3]{y/2}\,dy - 2\int_0^{0.39} \sqrt[3]{y/2}\,dy \\
&= 2\int_{0.39}^{0.4} \sqrt[3]{y/2}\,dy \approx 0.011m^2 = 11L
\end{aligned}$$

Plans for the Future

For next time:

WeBWork 8.4 and actively read section 8.4