Welcome to MAT136 LEC0501 (Assaf)

We continue to solve $\int \sqrt{\tan(x)}$. We substitute $w = s^2 - \sqrt{2}s + 1$ to get:

$$\int \frac{s}{s^2-\sqrt{2}s+1}ds = \int \frac{2s-\sqrt{2}}{2(s^2-\sqrt{2}s+1)} + \frac{1}{\sqrt{2}(s^2-\sqrt{2}s+1)}ds$$

$$= \int \frac{1}{2w}dw + \int \frac{1}{\sqrt{2}(s^2-\sqrt{2}s+1)}ds$$

$$= \frac{\log(w)}{2} + \frac{1}{\sqrt{2}}\int \frac{1}{(s-\frac{1}{\sqrt{2}})^2+\frac{1}{2}}$$

The last integral is computed using an inverse trig substitution. This is covered in chapter 7.4

Areas and Volumes – Slice 'em, Dice 'em, Integrate 'em

Assaf Bar-Natan

"Where trouble melts like lemon drops
High above the chimney top
That's where you'll find me"

–"Somewhere Over The Rainbow", Israel Kamakawiwo'ole

March 4, 2020

No Correct Answer

You are given a lemon, a knife, a piece of string, and a ruler. How would you use these tools to estimate the volume of the lemon? (Hint: the lemon can be destroyed in the process.)

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7 hours ago

1: Slice up the lemon into many equal-width(height) slices.
2: Use the ruler to measure the height and the radius of each lemon slice.
3: Use these measurements to find the volume of each slice.
4: Add all volumes together.

Comments 0 7

Hidden

7 hours ago

1. Slice the lemon in half with the knife
2. Measure the circumference of the lemon with the string
3. Use the ruler to find the radius for that circumference

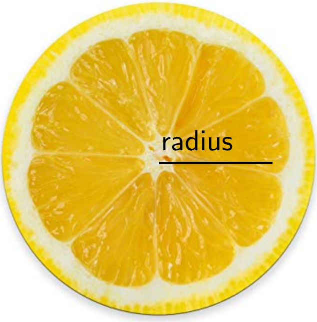
148/148 answered

Resume

100%

Cutting a Lemon

"When life gives you lemons, cut them up, and compute their volume"



We can estimate the volume by slicing the lemon into 1*cm*-thick slices. Then:

$$\text{Vol} = \sum_{\text{slices}} \text{Area(slice)} \times 1cm$$

$$\text{Area(slice)} = \pi \times \text{radius}^2$$

The smaller the slices, the better the approximation.

A Stack of Post-Its

Sometimes, shapes that look irregular, have nice-looking cross-sections:

Q: A $3\mathrm{in}$ tall stack of post-its is twisted like in the picture. What is the volume of the stack?

A Stack of Post-Its

Sometimes, shapes that look irregular, have nice-looking cross-sections:

Q: A 3in tall stack of post-its is twisted like in the picture. What is the volume of the stack?

A: Separate the stack into n individual post-its, each having a height of $\frac{3}{n}\text{in}$. The total volume is:

$$\sum_{i=1}^{n} l \times w \times \frac{3}{n}$$

Takeaway

In the same way that we sliced regions into small rectangles to compute areas using integrals, we can slice solids into thin cross-sections to compute volumes using integrals.

The Cat's Nest

The cats are burrowing into the top of a square hay-bale. The hay bale has a width of 18", a length of 36", and a height of 14". The cats burrow a cavity from the top whose radius is changing with the height above the ground. The radius of the cavity is $\frac{\sqrt{h}}{3}$ feet, where h is measured in feet above the ground.

How much hay is in the bale?

Submissions Closed

What are the steps we must take in order to use the "slicing method" to find the volume of an object?

✓ **20%** Answered Correctly

Correct Order

- **A** Draw a picture of the object and decide which direction to slice it in
- **F** Estimate the volume of each slice
- **B** Approximate the total volume by adding up the slices
- **C** Take the limit to obtain the exact value of the shape's volume
- **E** Interpret the limit as a Riemann sum then interpret the Riemann sum as an integral
- **D** Compute the definite integral to find the volume of the solid

March 3 at 11:25 PM results

Condense Text

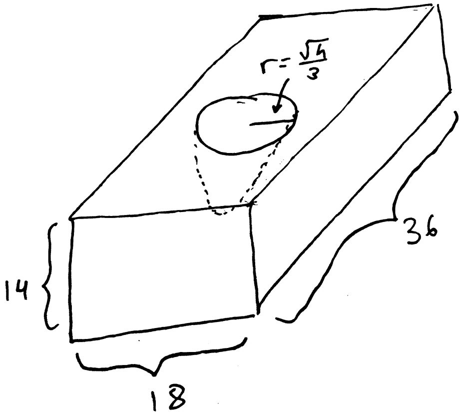
183/183 answered

Ask Again

Open | Closed | Responses | Correct

100%

Draw a Picture



Submissions Closed

In which direction should we slice our shape in order to find the volume?

✓ 61% Answered Correctly

A	Vectical slices (like the lemon)	16
B	Horizontal slices (like the post-its)	102
C	Both will work	50

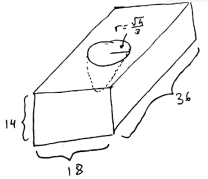
March 3 at 11:41 PM results | Segment Results | Compare with session | Show percentages | Hide Graph | Condense Text

168/169 answered | Ask Again

Open | Closed | Responses | ✓ Correct | 100%

No Correct Answer

Draw the horizontally-sliced cross-sections of the shape of the hay.



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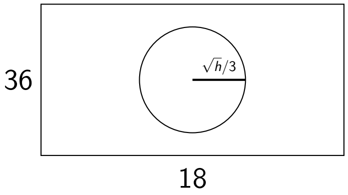
Comments 0 10

152/152 answered

Resume

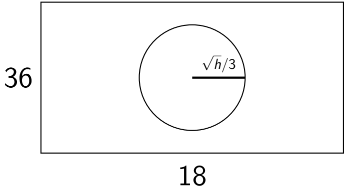
100%

Approximating the Volume



If the height of each cross-section is Δh, then the volume of the cross section at height h is:

Approximating the Volume



If the height of each cross-section is Δh, then the volume of the cross section at height h is:

$$V(h) = (18 \times 36 - \pi r^2)\Delta h$$
$$= 648\Delta h - \frac{\pi h}{9}\Delta h$$

Approximating the Total Volume

If the height of each cross-section is Δh, then the volume of the cross section at height h is:

$$\begin{aligned}\text{Cross-Sec. Vol} &= (18 \times 36 - \pi r^2)\Delta h \\ &= 648\Delta h - \frac{\pi h}{9}\Delta h\end{aligned}$$

Q: Write a Riemann sum that estimates the total volume (take $\Delta h = \frac{14}{n}$).

Approximating the Total Volume

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Q: Write a Riemann sum that estimates the total volume (take $\Delta h = \frac{14}{n}$).

$$\sum_{i=1}^{n} 648\frac{14}{n} - \frac{14\pi i}{9n}\frac{14}{n}$$

Taking the Limit

We've replaced h with $h_i = \frac{14}{n} i$, and $\Delta h = \frac{14}{n}$, then added it up. All that's left (in cubic inches) is to take the limit:

$$Vol = \lim_{n\to\infty} \sum_{i=1}^{n} 648\frac{14}{n} - \frac{14\pi i}{9n}\frac{14}{n}$$

Submissions Closed

Which of the following integrals corresponds to the Riemann sum $\lim_{n\to\infty}\sum_{i=1}^{n-1} 648\frac{14}{n} - \frac{14\pi i}{9n}\frac{14}{n}$

✓ **70%** Answered Correctly

A	$\int_0^{14}(648h - \frac{\pi}{9}h)dh$	11
B	$\int_0^{14}(648 - \frac{\pi}{9})dh$	20
C	$\int_0^{14}(648h - \frac{\pi}{9})dh$	21
D	$\int_0^{14}(648 - \frac{\pi}{9}h)dh$	125
E	$\int_0^{14}(648h - \frac{\pi}{9}h^2)dh$	1

March 3 at 11:59 PM results | Segment Results | Compare with session | Show percentages | Hide Graph | Condense Text

178/178 answered | Ask Again

Open | Closed | Responses | ✓ Correct | 72%

Evaluating the Integral

When all is said and done, the volume of hay left is:

$$\int_0^{14} \left(648 - \frac{\pi}{9}h\right) dh$$

Which is around $9000\mathrm{in}^3$, or, around 5.2 cubic feet.

Plans for the Future

For next time:

WeBWork 8.2 and actively read section 8.2

Ban cars on campus