Welcome to MAT136 LEC0501 (Assaf)

Last time, we had a trick: if $s = \sqrt{\tan(x)}$, then:

$$\int \sqrt{\tan(x)}dx = \int \frac{1}{\sqrt{2}}\left(\frac{s}{s^2 - \sqrt{2}s + 1} - \frac{s}{s^2 + \sqrt{2}s + 1}\right) ds$$

We work on the first term (the second is similar):

$$\int \frac{s}{s^2 - \sqrt{2}s + 1} ds = \int \frac{2s - \sqrt{2}}{2(s^2 - \sqrt{2}s + 1)} + \frac{1}{\sqrt{2}(s^2 - \sqrt{2}s + 1)} ds$$

You already know how to compute the first term here!

S11.8 Part 2 – The Perils of: Phase Diagrams, War, and Modeling

Assaf Bar-Natan

" I fought the war but the war won't
stop for the love of god.
I fought the war but the war won"

–"Monster Hospital", Metric

Feb. 28, 2020

The SIR Model – Contact Number

$$\frac{dS}{dt} = -\alpha SI$$
$$\frac{dI}{dt} = \alpha SI - \beta I$$

So:

$$\frac{dI}{dS} = \frac{\alpha SI - \beta I}{-\alpha SI} = -1 + \frac{\beta}{\alpha}\frac{1}{S}$$
$$= -1 + \frac{1}{cS}$$

Where we define $c = \frac{\alpha}{\beta}$, the contact number.

Why Contact Numbers

Parameter	What does is Measure?	Units	Interpretation
α	Spreadability		Fraction of S who are infected, per sick person per day.
β	Removal rate		Percent of I that get better per day
$\frac{1}{\beta}$			
c			

Why Contact Numbers

Parameter	What does is Measure?	Units	Interpretation
α	Spreadability	$\frac{1}{t \times ppl}$	Fraction of S who are infected, per sick person per day.
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$\frac{1}{\beta}$		t	
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$\frac{1}{\beta}$		t	Average amount of time someone is sick
c		$\frac{1}{ppl}$	Fraction of S that are infected per sick person

Takeaway

c is a measure of "contagion". It's a quantity that determines how many healthy people a sick person infects, all things considered.

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WARNING: some models use $s = \frac{S}{N}$, some models use different constants!

Match real-life scenarios

1:00 Hide Correct Answer

For each scenario on the left, match the constant or quantity that is REDUCED when the scenario happens

All results

Correct Order

	Scenario			Quantity	
1	Public transit is closed down	→	B	c	33
2	Infected individuals wear respirators	→	A	α	28
3	A vaccine is discovered and used	→	D	S	75
4	A cure is found	→	E	I	54
5	Better hospitals are built	→	C	$\frac{1}{\beta}$	47

UofT Model

You, too, can play with the parameters of the SIR model:

`https://art-bd.shinyapps.io/nCov_control/`

END (ESC)

No Correct Answer

Why is our model (the SIR model) imperfect? List three reasons.

Reply

Ordered by Most Liked

Shankavy Paramanathan — a day ago

1.some people are naturally immune and will not count as susceptible
2. R represents both dead and recovered and we cannot determine the survivors
3.immigrating emigrating

Comments 0 · 3

Miguel Weerasinghe — a day ago

1.yo my dude
2.acing this shizz
3.hold my beer
nah for serious likely dude to the fact that geographics and barrierss arent accounted for. epidemiology bros

Comments 0 · 3

XINYU ZHANG — a day ago

93/93 answered

Resume · 100%

Discussion: Why is our model imperfect?

- Changes in policy
- Constants are not actually constant
- Demographics are different
- Vaccines, medications
- ...

Cat Fight!

Marzipan and Rainbow are having a fight, and they brought all the other cats into it. They muster up their armies, and fight at midnight. Let $R(t)$ be the number of cats remaining in Rainbow's army, t minutes after midnight. Define $M(t)$ similarly.

We apply Lanchester's model:

$$\frac{dR}{dt} = -0.5M(t)$$

$$\frac{dM}{dt} = -0.3R(t)$$

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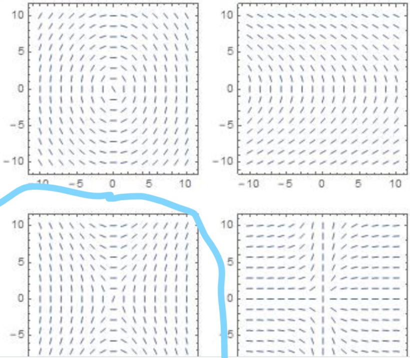
"I don't care how long the battle takes, I just want to win."

–Marzipan

Submissions Closed

Which of the following might be the slope field for $\frac{dR}{dM}$? Hint: compute $\frac{dR}{dM}$



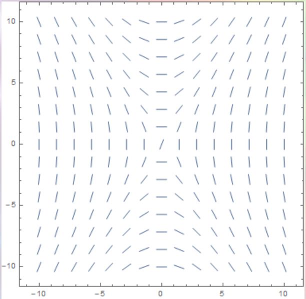
142/143 answered

Ask Again

Open | Closed | Responses | Correct

100%

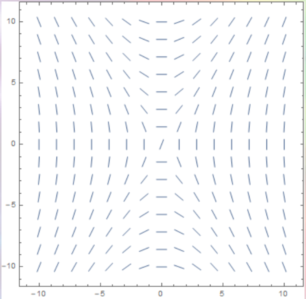
$$\frac{dR}{dM} = \frac{0.5}{0.3}\frac{M}{R}$$



An **Equilibrium point** is a point where:

$$\frac{dR}{dt} = 0$$
$$\frac{dM}{dt} = 0$$

$$\frac{dR}{dM} = \frac{0.5}{0.3}\frac{M}{R}$$



An **Equilibrium point** is a point where:

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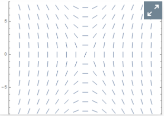
$$\frac{dM}{dt} = 0$$

Q: Does there exist an equilibrium point for this system of differential equations? Yes! At $R = 0, M = 0$

Submissions Closed

Both Rainbow and Marzipan bring five cats to the fight. Who wins?



✓ **55%** Answered Correctly

A	Rainbow	25
B	Marzipan	67
C	It's a tie	19
D	We'd need to solve the system equations explicitly to find out	11

February 28 at 12:03 AM results · Segment Results · Compare with session · **Show percentages** · **Hide Graph** · **Condense Text**

122/123 answered · Ask Again

Open · Closed · Responses · ✓ Correct · 100%

Takeaway

We don't need to solve the differential equation! The slope field can tell us quite a bit!

For more: see the SIR model example in the text.

Plans for the Future

For next time:
Review Taylor polynomials!