# Welcome to MAT135 LEC0501 (Assaf)

As you come in, introduce yourself to someone you haven't met yet.

# S5.1&5.2 – Riemann Sums, Erors, and Areas

Assaf Bar-Natan

" In the morning I'd awake
And I couldn't remember
What is love and what is hate
The calculations error "

–" In The Morning of the Magicians ", The Flaming Lips

Jan. 8, 2020

# Announcements

- Read the syllabus (it's on Quercus).
- WeBWork is due the night before class
- We do not answer e-mails sent via WeBWork
- TopHat is graded by participation only. If it becomes meaningless, this will change!

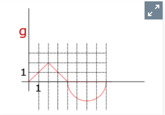
# Integrals and Areas

In your groups, write a sentence explaining the geometric interpretation of the expression:

$$\int_a^b f(x)dx$$

Submissions Closed

The function $g$ is drawn below. What is $\int_0^6 g(x)\,dx$? (give answer up to two decimal places)




✓ 42% Answered Correctly

| | |
|---|---|
| 0.7584 to 0.9584 | 77 |
| -21.2416 to -21.0416 | 1 |
| -8.6416 to -8.4416 | 7 |
| -7.2416 to -7.0416 | 1 |
| -4.8416 to -4.6416 | 1 |

January 7 at 10:39 PM results

Show percentages Hide Graph Condense Text

182/182 answered

Ask Again

Open Closed Responses Correct

100%

# Takeaway

**The integral of a function between $a$ and $b$ is the signed area between the function and the $x$-axis.**

Submissions Closed

Let $f(x) = \log(\log(x))$. Then the integral $\int_3^5 f''(x)\,dx$ is

✓ **67%** Answered Correctly

| | | |
|---|---|---|
| A | Positive, and I'm confident in my answer. | 18 |
| B | Positive, and I'm not confident in my answer. | 32 |
| C | Negative, and I'm not confident in my answer. | 58 |
| D | Negative, and I'm confident in my answer. | 70 |
| E | I have no idea. | 12 |

January 7 at 10:42 PM results · Segment Results · Compare with session · Show percentages · Hide Graph · Condense Text

190/190 answered · Ask Again

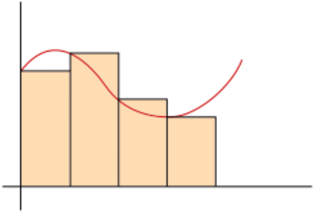
Open · Closed · Responses · ✓ Correct · 100%

# Takeaway

**The fundamental theorem can allow us to compute hard integrals in an instant. We just need to identify them as derivatives!**

# Computing Integrals – An Idea

- Draw the function
- Divide the interval
- Pick left- or right-rectangles
- Add up areas



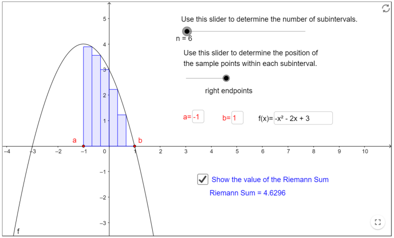
How does this work in practice?

# Playing with Geogebra

In groups, spend five minutes playing around with the applet:

`https://www.geogebra.org/m/xJsZTG2i`

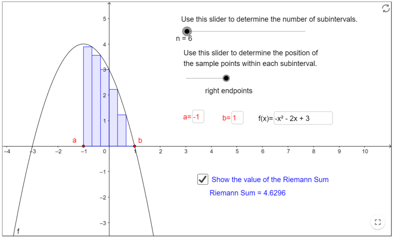
# Playing with Geogebra




For $n = 6$, the right Riemann sum is $(\Delta t = \frac{1}{3})$:

$$\Delta t(f(-\frac{2}{3}) + f(-\frac{1}{3}) + f(0) + f(\frac{1}{3}) + f(\frac{2}{3}) + f(1))$$

# Playing with Geogebra

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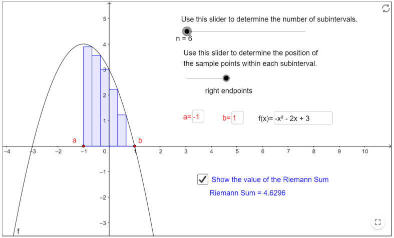
$$\Delta t(f(-\frac{2}{3}) + f(-\frac{1}{3}) + f(0) + f(\frac{1}{3}) + f(\frac{2}{3}) + f(1))$$

What is the left Riemann sum?

# Playing with Geogebra

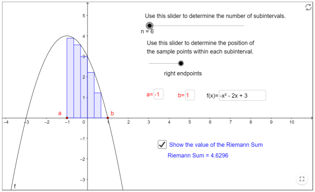



The integral is somewhere between the left and right Riemann sums:

$$\_\_\_\_\_ \leq \int_{-1}^{1} (-x^2 - 2x + 3)dx \leq \_\_\_\_\_$$

Which Riemann sum goes where?

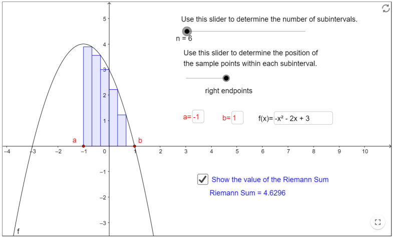
# Playing with Geogebra




$$R.H.S \leq \int_{-1}^{1} (-x^2 - 2x + 3)dx \leq L.H.S$$

Rainbow the cat wants to compute the area under the curve using a left-Riemann sum. He wants to know how far away from the true area his computation be.

# Playing with Geogebra




We know:

$$R.H.S = \Delta t(f(-\frac{2}{3}) + f(-\frac{1}{3}) + f(0) + f(\frac{1}{3}) + f(\frac{2}{3}) + f(1))$$

$$L.H.S = (f(-1) + f(-\frac{2}{3}) + f(-\frac{1}{3}) + f(0) + f(\frac{1}{3}) + f(\frac{2}{3}))$$

What is $L.H.S - R.H.S$?

# Playing with Geogebra

**Q:** Rainbow wants to compute the area under the curve $-x^2 - 2x + 3$ between $x = -1$ and $x = 1$. He wants his computation to fall within 0.02 of the true value. How many rectangles does He need?

# Playing with Geogebra

**Q:** Rainbow wants to compute the area under the curve $-x^2 - 2x + 3$ between $x = -1$ and $x = 1$. He wants his computation to fall within 0.02 of the true value. How many rectangles does He need?

**A:** We know that the maximal error is $L.H.S - R.H.S$, which is given by $\Delta t(f(-1) - f(1))$. Plugging in values, we want:

$$0.02 \geq \Delta t \cdot 4$$

**Q:** Rainbow wants to compute the area under the curve $-x^2 - 2x + 3$ between $x = -1$ and $x = 1$. He wants his computation to fall within 0.02 of the true value. How many rectangles does He need?

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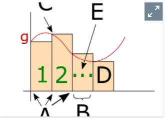
We know $\Delta t = \frac{2}{n}$, so to make $\Delta t < 0.005$, we need $n$ to be at least 400.

# Takeaway

**When a function is monotonic, we have a good way to estimate the error between the left- and the right- Riemann sums**

Submissions Closed

In the picture below, match the letter to the term in the expression: $\lim_{n\to\infty} \sum_{i=0}^{n-1} g(t_i)\Delta t$




✓ 2% Answered Correctly

Correct Order

| | | | | | |
|---|---|---|---|---|---|
| 1 | A | → | F | $t_i$ | 59 |
| 2 | B | → | A | $\Delta t$ | 90 |
| 3 | C | → | E | $g(t_1)$ | 91 |
| 4 | D | → | C | $n$ | 20 |

January 7 at 10:14 PM results

Condense Text

167/167 answered

Ask Again

Open | Closed | Responses | ✓ Correct

100%

# One-Minute Explanation

Write a sentence explaining what happens to the left- and right-Riemann sums when we take the limit as $n \to \infty$.

# One-Minute Explanation

Write a sentence explaining what happens to the left- and right-Riemann sums when we take the limit as $n \to \infty$.

“ **When we take the limit as $n \to \infty$, the left and the right Riemann sums converge to the same thing. This is the signed area under the function, or, the definite integral.**”

# Plans for the Future

For next time:
**WeBWork 5.3 and read section 5.3**