Welcome to MAT136 LEC0501 (Assaf)

Weather is finally nice! How've you been enjoying it?

S11.8 Systems of ODE's and The SIR Model (Part 1)

Assaf Bar-Natan

" For there's Basie, Miller, Satchmo
And the king of all, Sir Duke
And with a voice like Ella's ringing out
There's no way the band can lose"

–"SIR Duke", Stevie Wonder

Feb. 26, 2020

The SIR Model

Reminder: The SIR model says:

$$\frac{dS}{dt} = -\alpha SI$$
$$\frac{dI}{dt} = \alpha SI - kI$$
$$\frac{dR}{dt} = kI$$

We used k, the textbook uses β.

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We used k, the textbook uses β.
Q: Remind yourself what S, I, and R mean in the SIR model.

Finding Values for β and α

Get into groups of three or four, and open up a spreadsheeting program.

- Title the first column: DATA

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- Navigate to: `https://covid2019.azurewebsites.net`, and explore the data on the bottom bar of the site

Finding Values for β and α

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- Title the first column: DATA
- Navigate to: `https://covid2019.azurewebsites.net`, and explore the data on the bottom bar of the site
- For Hubei, copy down $I(t)$ into the first column of a spreadsheet (use only the data from the first 15-16 days)

Submissions Closed

What is your best estimate for β (or k, if we are not using the textbook) in applying the SIR model to the coronavirus in Hubei? Round to one significant digit.

✓ 1% Answered Correctly

0.04	2
1.388	1
105	1
-105	1
1.4	1
123	1

February 26 at 2:02 AM results

Show percentages Hide Graph Condense Text

163/163 answered

Ask Again

Open Closed Responses ✓ Correct

100%

Takeaway

β **is easily measured as the death and recovery rate**

Making a Model

In your groups:

- Make a new column in the spreadsheet. Label it S
- Make a new column in the spreadsheed. Label it I
- Make a new column in the spreadsheet. Label it R
- What should $S(1)$ be? What should $R(1)$ be?

We will next use Euler's method to fill in the rest of the model.

Making a Model

In your groups:

- Make a new column in the spreadsheet. Label it S
- Make a new column in the spreadsheed. Label it I
- Make a new column in the spreadsheet. Label it R
- What should $S(1)$ be? What should $R(1)$ be? $S(1)$ is the population of Hubei, $R(1) = 0$

We will next use Euler's method to fill in the rest of the model.

Making a Model

- Write a formula for $I(2)$, $S(2)$, and $R(2)$ involving $S(1)$, $I(1)$, $R(1)$, the constant $\beta = 0.04$, and an unknown constant, α (maybe start by plugging in $\alpha = 0.000001$.)
- Extend the formula down (click and drag) to predict $I(t)$, $S(t)$, and $R(t)$. **Note: they will have to depend on each other!**
- Do your predictions match the data column? What parameter should you change?

Hint: $I(t+1) \approx I(t) + I'(t)$

Takeaway

We can use a spreadsheet and Euler's method to solve an ODE, and to make predictions

Interpreting the Constants

When we developed the SIR model:

- α represented the infection rate per sick person per day.
- $k = \beta$ represented the rate at which people recovered.

Go back in your notes, or to lecture 14, and remind yourself how we used these interpretations to derive the SIR model.

Interpreting the Constants

When we developed the SIR model:

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Now: $\frac{1}{k}$ can also be interpreted as the average amount of time a person is sick with the virus.

Interpreting the Constants

When we developed the SIR model:

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How can we use units to understand this interpretation? What are the units of k? What are the units of $\frac{1}{k}$?

Phase-Plane Introduction

We use the chain rule:

$$\frac{dI}{dS} = \frac{\frac{dI}{dt}}{\frac{dS}{dt}}$$

This, along with the SIR model equations, allows us to solve for I in terms of S.

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This, along with the SIR model equations, allows us to solve for I in terms of S.

In your groups, write $\frac{dI}{dS}$ exclusively in terms of S, α, and β (or k).

Submissions Closed

In Hubei, assume that the contact number is approximately $\frac{1}{6,000,000}$. At what value of S will I be maximal?

✓ 12% Answered Correctly

6000000	14
67	1
-1	1
75	1
0	23
100	2

February 26 at 2:06 AM results

Show percentages Hide Graph Condense Text

120/121 answered

Ask Again

Open Closed Responses Correct 100%

Takeaways

$c = \frac{1}{6{,}000{,}000}$ means that on average, an infected person has close contact with about $\frac{1}{6{,}000{,}000}$th of the population of Hubei. This is around 10 people, which is quite reasonable.

The constant c, is called the contact number, and next time, we will see how it can be used to help prevent an epidemic.

Plans for the Future

For next time:
actively read section 11.8