

# Welcome to MAT136 LEC0501 (Assaf)



Weather is finally nice! How've you been enjoying it?



## S11.8 Systems of ODE's and The SIR Model (Part 1)

Assaf Bar-Natan

“ For there's Basie, Miller, Satchmo  
And the king of all, Sir Duke  
And with a voice like Ella's ringing out  
There's no way the band can lose”

–“SIR Duke”, Stevie Wonder

Feb. 26, 2020

# The SIR Model

Reminder: The SIR model says:

$$\frac{dS}{dt} = -\alpha SI$$

$$\frac{dI}{dt} = \alpha SI - kI$$

$$\frac{dR}{dt} = kI$$

We used  $k$ , the textbook uses  $\beta$ .

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**Q:** Remind yourself what  $S$ ,  $I$ , and  $R$  mean in the SIR model.

## Finding Values for $\beta$ and $\alpha$




**Get into groups of three or four, and open up a spreadsheeting program.**

- Title the first column: DATA



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- Navigate to: <https://covid2019.azurewebsites.net>, and explore the data on the bottom bar of the site
- For Hubei, copy down  $I(t)$  into the first column of a spreadsheet (use only the data from the first 15-16 days)



Submissions Closed

What is your best estimate for  $\beta$  (or  $k$ , if we are not using the textbook) in applying the SIR model to the coronavirus in Hubei? Round to one significant digit.

✓ 1% Answered Correctly

0.04		2
1.388		1
105		1
-105		1
1.4		1
123		1

February 26 at 2:02 AM results ▾

[Show percentages](#) [Hide Graph](#) [Condense Text](#)

163/163 answered

[Ask Again](#)

⏪ ⏩ ⏴ ⏵ ⏶ ⏷ ⏸ ⏹ ⏺ ⏻ ⏼ ⏽ ⏾ ⏿

🔍 100% ⚙️



# Takeaway



**$\beta$  is easily measured as the death and recovery rate**

# Making a Model



## In your groups:

- Make a new column in the spreadsheet. Label it S
- Make a new column in the spreadsheet. Label it I
- Make a new column in the spreadsheet. Label it R
- What should  $S(1)$  be? What should  $R(1)$  be?

We will next use Euler's method to fill in the rest of the model.



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- Make a new column in the spreadsheet. Label it I
- Make a new column in the spreadsheet. Label it R
- What should  $S(1)$  be? What should  $R(1)$  be?  $S(1)$  is the population of Hubei,  $R(1) = 0$

We will next use Euler's method to fill in the rest of the model.

# Making a Model

- Write a formula for  $I(2)$ ,  $S(2)$ , and  $R(2)$  involving  $S(1)$ ,  $I(1)$ ,  $R(1)$ , the constant  $\beta = 0.04$ , and an unknown constant,  $\alpha$  (maybe start by plugging in  $\alpha = 0.000001$ .)
- Extend the formula down (click and drag) to predict  $I(t)$ ,  $S(t)$ , and  $R(t)$ . **Note: they will have to depend on each other!**
- Do your predictions match the data column? What parameter should you change?

*Hint:  $I(t + 1) \approx I(t) + I'(t)$*



**We can use a spreadsheet and Euler's method to solve an ODE, and to make predictions**

# Interpreting the Constants

When we developed the SIR model:

- $\alpha$  represented the infection rate per sick person per day.
- $k = \beta$  represented the rate at which people recovered.

Go back in your notes, or to lecture 14, and remind yourself how we used these interpretations to derive the SIR model.

# Interpreting the Constants

When we developed the SIR model:

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**Now:**  $\frac{1}{k}$  can also be interpreted as the average amount of time a person is sick with the virus.

How can we use units to understand this interpretation? What are the units of  $k$ ? What are the units of  $\frac{1}{k}$ ?



# Phase-Plane Introduction

We use the chain rule:

$$\frac{dI}{dS} = \frac{\frac{dI}{dt}}{\frac{dS}{dt}}$$

This, along with the SIR model equations, allows us to solve for  $I$  in terms of  $S$ .

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This, along with the SIR model equations, allows us to solve for  $I$  in terms of  $S$ .

In your groups, write  $\frac{dI}{dS}$  exclusively in terms of  $S$ ,  $\alpha$ , and  $\beta$  (or  $k$ ).

Submissions Closed

In Hubei, assume that the contact number is approximately  $\frac{1}{6,000,000}$ . At what value of  $S$  will  $I$  be maximal?

✓ 12% Answered Correctly

6000000	█	14
67		1
-1		1
75		1
0	█	23
100		2

February 26 at 2:06 AM results ▾

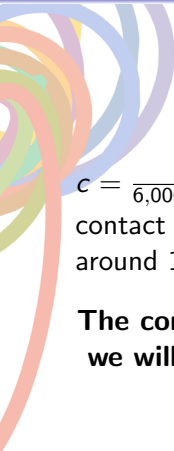
Show percentages Hide Graph Condense Text

120/121 answered

Ask Again

Q 100% ⚙

# Takeaways



$c = \frac{1}{6,000,000}$  means that on average, an infected person has close contact with about  $\frac{1}{6,000,000}$ th of the population of Hubei. This is around 10 people, which is quite reasonable.

**The constant  $c$ , is called the contact number, and next time, we will see how it can be used to help prevent an epidemic.**



For next time:  
**actively read section 11.8**