Welcome to MAT136 LEC0501 (Assaf)

Over reading week, did you do something:

- Fun?
- Hard?
- Rewarding?

S11.5 – Growth Models

Assaf Bar-Natan

> “ Now, for ten years we’ve been on our own
> And moss grows fat on a rolling stone
> But, that’s not how it used to be”
>
> –“American Pie”, Don McLean

Feb. 24, 2020

Game Plan

- Today: section 11.5
- Wednesday & Friday: section 11.8
- New WeBWork: Taylor polynomials review “136TaylorSolutions”

Key Points Round Robin

Get into groups of three or four

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- As a group, come up with three big key ideas from this chapter.

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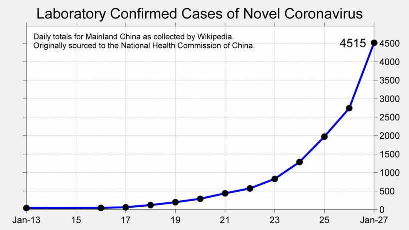
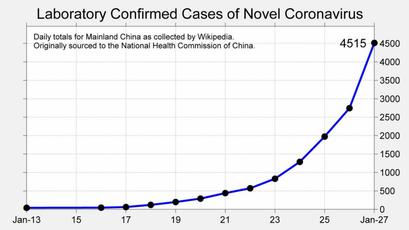
- As a group, come up with three big key ideas from this chapter.
- Pick a WeBWork problem from section 11.5. What key ideas does it relate to?

COVID-19 Growth



What function could model this data?

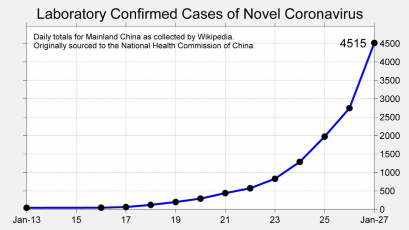
COVID-19 Growth



A reasonable guess:

$$I(t) = I_0 e^{kt}$$

COVID-19 Growth



A reasonable guess:

$$I(t) = I_0 e^{kt}$$

What value should we choose for k?

Possible Reasons for Discrepancy

- Data is imprecise
- S is approximately constant, so I' is approximately proportional to I
- The exponential model is not a good model to use in this case
- The data is not actually an exponential.

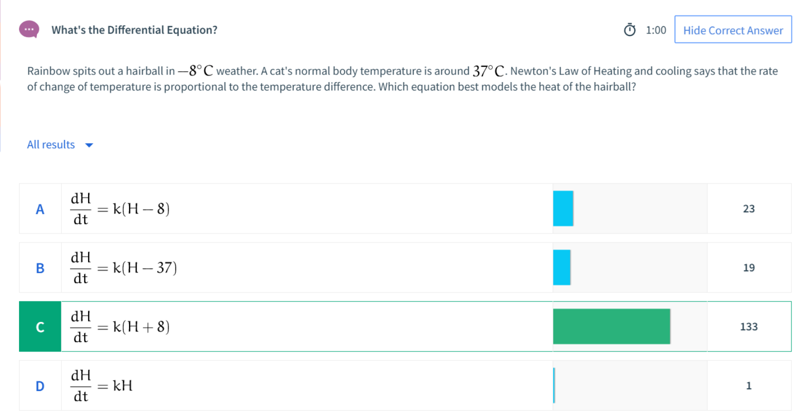
`https://www.worldometers.info/coronavirus/`

Takeaways

We can use a graph to track in real-time whether the SIS model is a good model

Punctuated Lecture: Rainbow's Hairball

Rainbow spits out a hairball in $-8^\circ C$ weather. A cat's normal body temperature is around $37^\circ C$. After one minute, the ball's temperature was $20^\circ C$. We will try to model the hairball's temperature as a function of time.



What's the Differential Equation?

Rainbow spits out a hairball in $-8^\circ C$ weather. A cat's normal body temperature is around $37^\circ C$. Newton's Law of Heating and cooling says that the rate of change of temperature is proportional to the temperature difference. Which equation best models the heat of the hairball?

A	$\frac{dH}{dt} = k(H-8)$	23
B	$\frac{dH}{dt} = k(H-37)$	19
C	$\frac{dH}{dt} = k(H+8)$	133
D	$\frac{dH}{dt} = kH$	1

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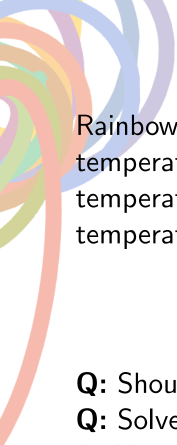

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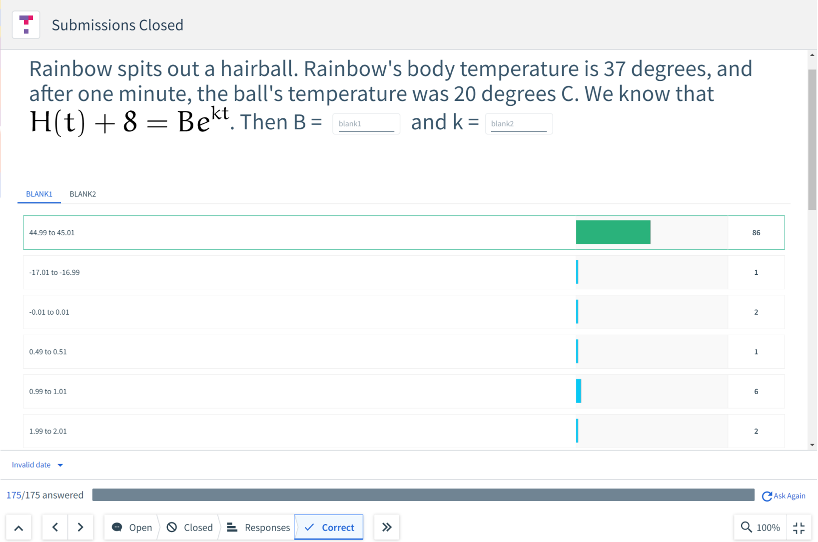
$$\frac{dH}{dt} = k(H + 8)$$

Q: Should k be positive of negative?

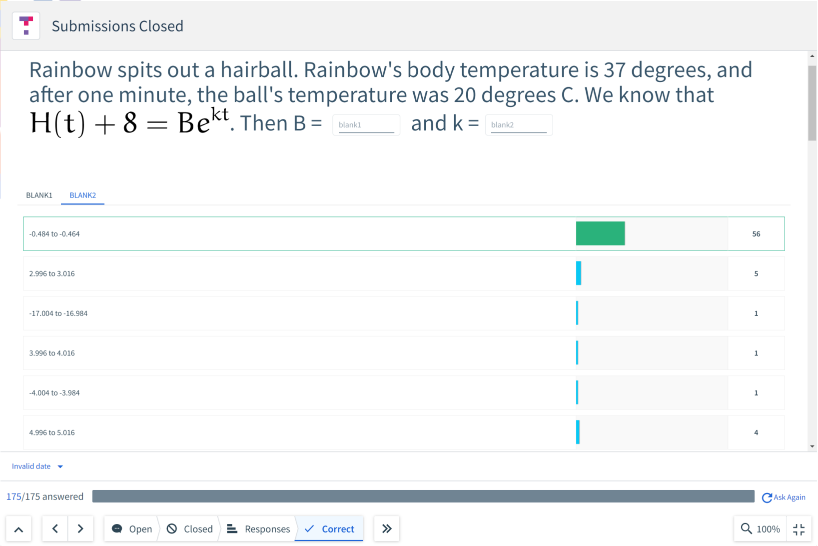
Q: Solve this differential equation.

A: Using separation of variables, $H(t) + 8 = Be^{kt}$.



Rainbow spits out a hairball. Rainbow's body temperature is 37 degrees, and after one minute, the ball's temperature was 20 degrees C. We know that $H(t)+8=Be^{kt}$. Then B = blank1 and k = blank2

44.99 to 45.01	86
-17.01 to -16.99	1
-0.01 to 0.01	2
0.49 to 0.51	1
0.99 to 1.01	6
1.99 to 2.01	2



Submissions Closed

Rainbow spits out a hairball. Rainbow's body temperature is 37 degrees, and after one minute, the ball's temperature was 20 degrees C. We know that $H(t) + 8 = Be^{kt}$. Then B = blank1 and k = blank2

BLANK1 BLANK2

-0.484 to -0.464	56
2.996 to 3.016	5
-17.004 to -16.984	1
3.996 to 4.016	1
-4.004 to -3.984	1
4.996 to 5.016	4

Invalid date

175/175 answered

Punctuated Lecture: Rainbow's Hairball

$$H(t) + 8 = Be^{kt}$$

We are given that $H(0) = 37$, so this means that $B = 45$.

Punctuated Lecture: Rainbow's Hairball

$$H(t) + 8 = Be^{kt}$$

We are given that $H(0) = 37$, so this means that $B = 45$. Moreover, we know $H(1) = 20$, so:

$$28 = 45e^{k}$$

giving us $k \approx -0.474$

Takeaway

We can use initial conditions and another point to find constants that give a particular solution to a heat-law-type problem

Equilibrium Points

Here's a totally wonky, and completely random differential equation:

$$\frac{dy}{dx} = (y-1)(y+1)$$

Q: What are its equilibrium solutions?

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Here's a totally wonky, and completely random differential equation:

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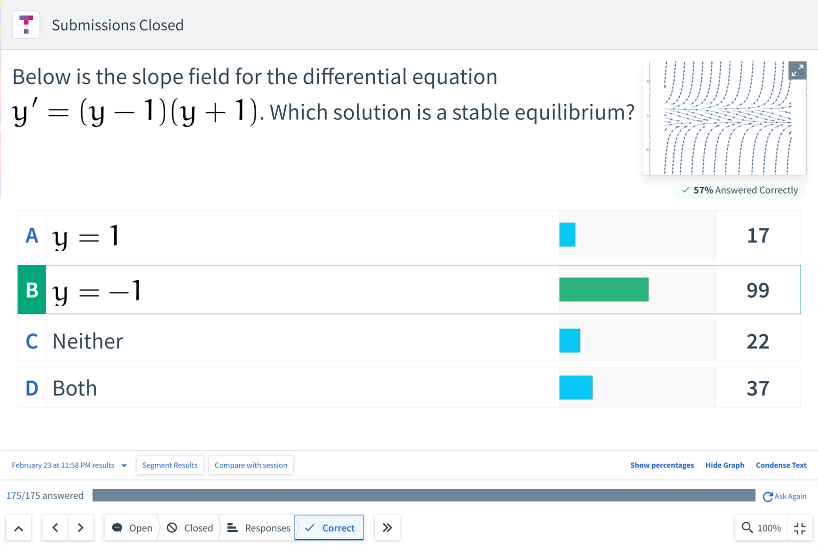
Q: What are its equilibrium solutions?
A: $y = 1$ and $y = -1$



Submissions Closed

Below is the slope field for the differential equation $y' = (y-1)(y+1)$. Which solution is a stable equilibrium?

✓ **57%** Answered Correctly

A	$y = 1$	17
B	$y = -1$	99
C	Neither	22
D	Both	37

175/175 answered

Takeaway

We can tell the difference between stable and unstable equilibria by looking at the slope fields.

Plans for the Future

For next time:

WeBWork 11.8 and actively read section 11.8