Welcome to MAT136 LEC0501 (Assaf)

Was the midterm what you expected? What surprised you? What would you change next time?

S11.4 – Separation of Variables – $\frac{dy}{dx}$ is Still not a Fraction

Assaf Bar-Natan

" How long, how long will I slide?
Separate my side, I don't
I don't believe it's bad"

–"Otherside", Red Hot Chili Peppers

Feb. 14, 2020

Ice Cream Sandwich

In your groups, share:

- A time you had a good success

Ice Cream Sandwich

In your groups, share:

- A time you had a good success
- A time you failed

Ice Cream Sandwich

In your groups, share:

- A time you had a good success
- A time you failed
- A time you recovered

What is Separation of Variables?

We wish to solve:

$$\frac{dy}{dx} = g(x)f(y)$$

Thinking of $\frac{dy}{dx}$ as a ratio (it's not), we get:

$$\int \frac{1}{f(y)} dy = \int g(x) dx$$

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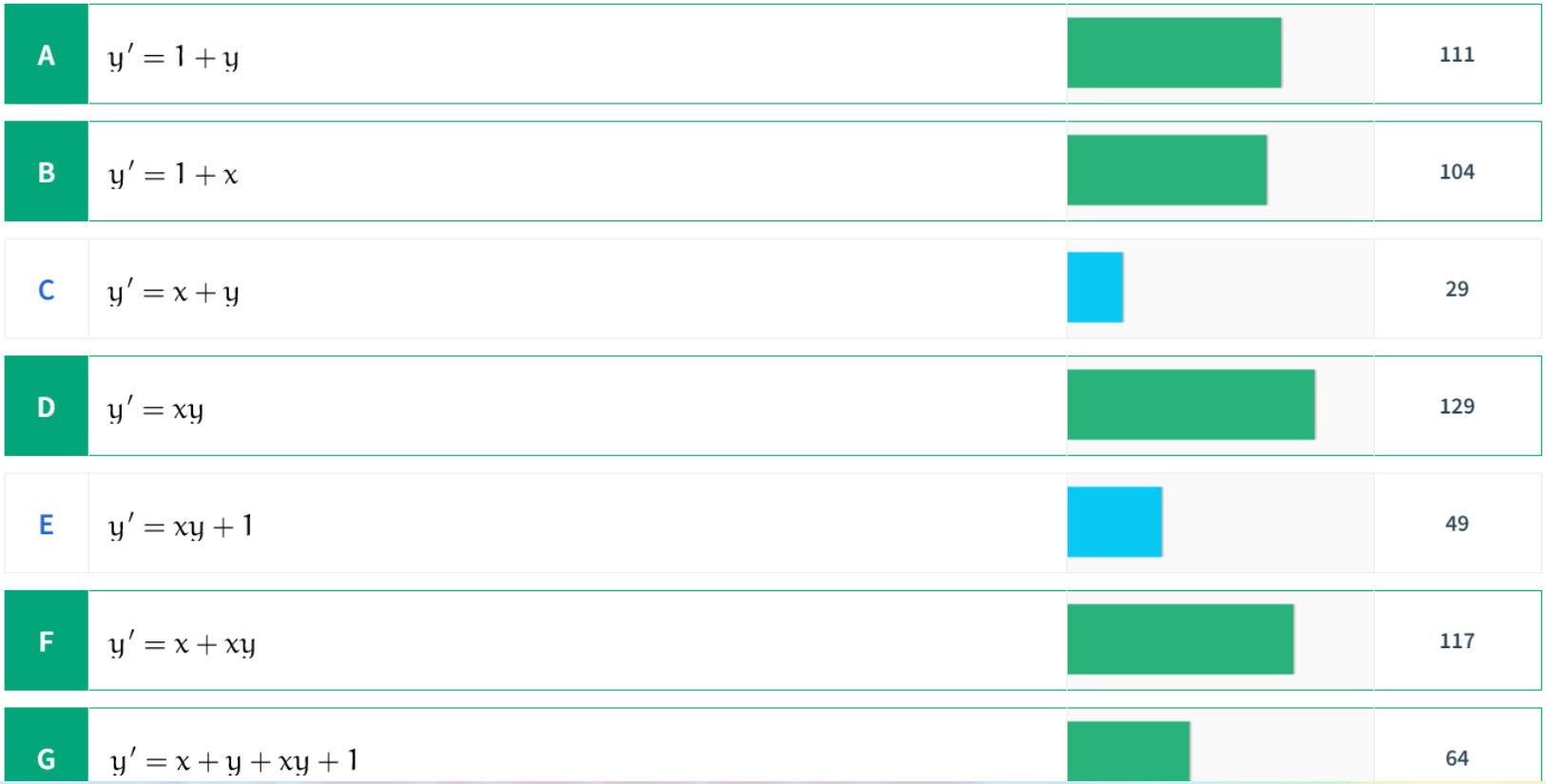
This gives us an relation between x and y, which is the solution to the differential equation

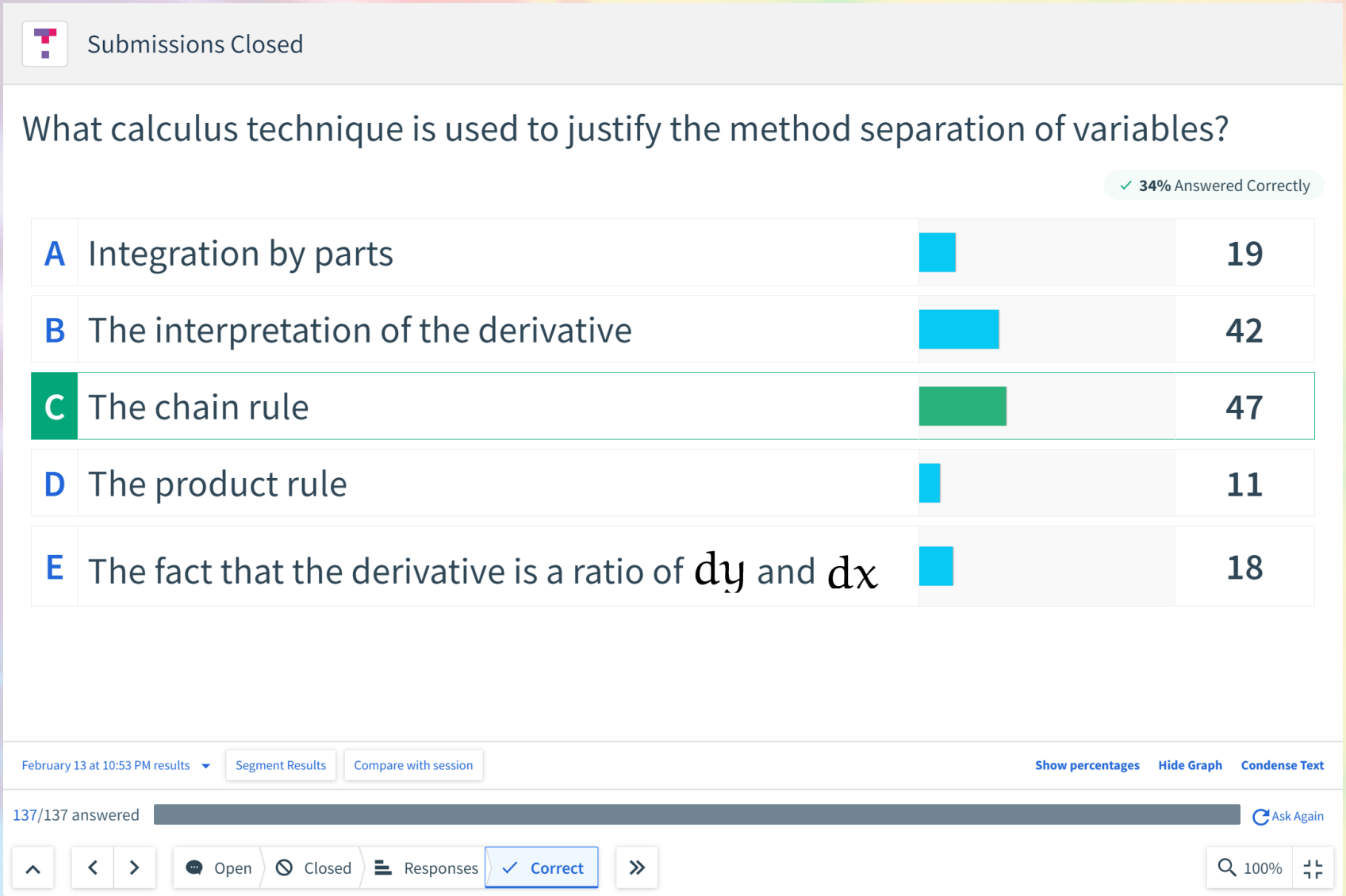
Which Equations?

Worth **1 participation point**and **0 correctness points**

Which of the following differential equations are separable? Click all that are separable

All results




Submissions Closed
What calculus technique is used to justify the method separation of variables?
34% Answered Correctly
A Integration by parts 19
B The interpretation of the derivative 42
C The chain rule 47
D The product rule 11
E The fact that the derivative is a ratio of dy and dx 18
February 13 at 10:53 PM results
Segment Results
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137/137 answered
Ask Again
Open
Closed
Responses
Correct
100%

Justification for Separation of Variables

A differential equation is called *separable* if it can be written in the form

$$\frac{dy}{dx} = g(x)\, f(y)\,.$$

Provided $f(y) \neq 0$, we write $f(y) = 1/h(y)$, so the right-hand side can be thought of as a fraction,

$$\frac{dy}{dx} = \frac{g(x)}{h(y)}.$$

If we multiply through by $h(y)$, we get

$$h(y)\,\frac{dy}{dx} = g(x)\,.$$

Thinking of y as a function of x, so $y = y(x)$, and $dy/dx = y'(x)$, we can rewrite the equation as

$$h(y(x)) \cdot y'(x) = g(x)\,.$$

Now integrate both sides with respect to x:

$$\int h(y(x)) \cdot y'(x)\ dx = \int g(x)\ \ dx\,.$$

The form of the integral on the left suggests that we use the substitution $y = y(x)$. Since $dy = y'(x)\,dx$, we get

$$\int h(y)\ dy = \int g(x)\ \ dx\,.$$

If we can find antiderivatives of h and g, then this gives the equation of the solution curve.

Takeawy

While $\frac{dy}{dx}$ is not a fraction, it can be useful to think of it as one.

The textbook is a useful resource!!!!!

Punctuated Lecture: The Cat Population

Last time, we modeled the population of cats by:

$$\frac{dy}{dt} = y(1 - y/30)$$

Question: Use separation of variables to write this differential equation as an equality of integrals.

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This is solved by:

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Question: Verify that

$$\log\left(\frac{y}{30 - y}\right)$$

is an antiderivative of $\frac{1}{y-y^2/30}$. (You may use a computer)

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Question: Write y as a function of t.

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This is solved by:

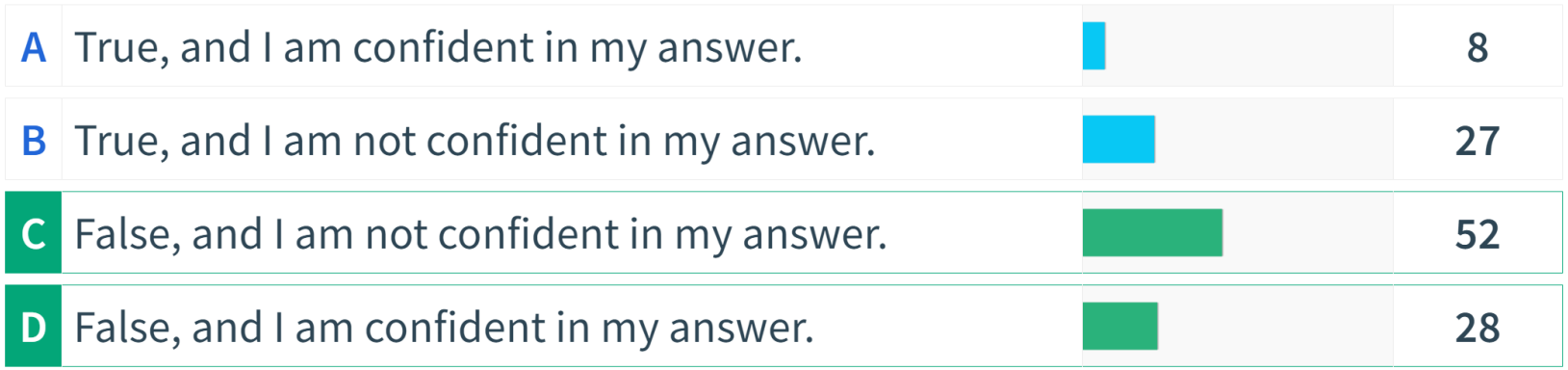
$$y(t) = \frac{30e^t}{1 + e^t}$$

Question: We earlier said that the number of cats at $t = 0$ was 20, but plugging in $t = 0$ above does not yield 20. What happened?

Submissions Closed

Using separation of variables to solve a differential equation, we can always get y as an explicit function of x

✓ **70%** Answered Correctly



Invalid date | Segment Results | Compare with session | Show percentages | Hide Graph | Condense Text

115/115 answered | Ask Again

Open | Closed | Responses | Correct | 100%

Separation of Variables – Practice

Solve the following differential equation using separation of variables:

$$y' = \frac{1}{1+y^4}$$

Takeaway

Separation of variables gives an implicit solution to the differential equation, not an explicit one

Plans for the Future

For next time:

WeBWork 11.5 and actively read section 11.5