#### Welcome to MAT135 LEC0501 (Assaf)

Last week, 
$$u = \tan(x)$$
, then  $s = \sqrt{u}$  gave us:

$$\int \sqrt{\tan(x)} dx = 2 \int \frac{s^2}{s^4 + 1} ds$$

Here's the trick:

$$2\int \frac{s^2}{s^4 + 1} ds = \int \frac{1}{\sqrt{2}} \left( \frac{s}{s^2 - \sqrt{2}s + 1} - \frac{s}{s^2 + \sqrt{2}s + 1} \right) ds$$

Next week: we'll compute one of these terms.

## S11.3 – Euler's Method – Stop, Point, Shoot, Repeat

#### Assaf Bar-Natan

" Eat, sleep, rave, repeat Eat, sleep, rave, repeat Eat, sleep, rave, repeat Eat, sleep, rave, repeat"

-"Eat, Sleep, Rave, Repeat", Fatboy Slim

Feb. 12, 2020

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Assaf Bar-Natan 2/14

Euler's method is a bit like a biathelon.



Myriam Bédard, Canadian gold medalist in Biathelon, 1994 Winter Olympics In a nutshell:

- Pick a starting point
- Use derivative to estimate change
- Move to next point
- Repeat

What does this mean? Let's assume that:

$$y'(t)=f(y,t)$$

This differential equation has a family of solutions. If we specify that our solution passes through (0,0), then we know:

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This lets us estimate  $y(0.01) \approx 0.01y'(0)$ , giving us a new point to start with.

#### Takeaway

#### To estimate the solution of a differential equation at a point, we can apply Euler's method

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#### Round Robin: WeBWork

In your groups, share a question in the WeBWork that you either found challenging or a question that you found interesting.

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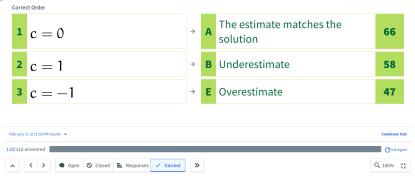
Why did we include this question in the WeBWork? What key point from the chapter does it relate to?

Submissions Closed

Below is pictured the slope field for some differential equation. For the initial condition y(1) = c, will Euler's method give an over- or an under-estimate when trying to estimate y(2)?







### Writing Exercise

Write a short paragraph explaining to the students who aren't here why Euler's method works, and how we can make it more precise.

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Write a short paragraph explaining to the students who aren't here why Euler's method works, and how we can make it more precise.

We can make Euler's method more precise by making the jumps smaller. That way, the estimate of the derivative is better.

The cats are reproducing! Their numbers are increasing! It's a happy time to be a cat. Let y(t) denote the number of cats t months after the start of the year, and assume that y'(t) = y(t)(1 - y(t)/30)(. Assume that y(0) = 20 Use Euler's method to estimate the number of cats after two months. Use 4 steps. (Hint: use a table)

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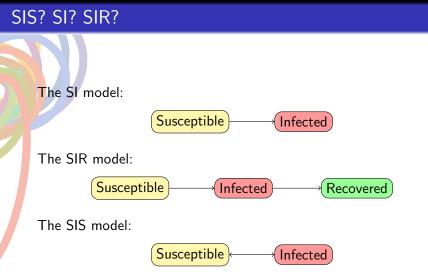
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#### Bonus: Chaos, Fractals, Dynamics

An interesting video related to this:

https://www.youtube.com/watch?v=ovJcsL7vyrk

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### Coronavirus – SIS? SI? SIR?

Go online, and search for information about the current Coronavirus outbreak. Which model is best? What searches did you make?

#### Coronavirus - SIS? SI? SIR?

Go online, and search for information about the current Coronavirus outbreak. Which model is best? What searches did you make? After infection, what happens to surviving patients?

# When modeling the Coronavirus, what model is best, considering what we know about it?

ASIS model31BSIR model53CSI model9



33% Answered Correctly

#### Plans for the Future

For next time: WeBWork 11.4 and actively read section 11.4

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