Welcome to MAT135 LEC0501 (Assaf)

Last week, $u = \tan(x)$

$$\int \sqrt{\tan(x)}dx = \int \frac{\sqrt{u}}{u^2+1}du$$

Now, substitute $s = \sqrt{u}$:

$$\int \frac{\sqrt{u}}{u^2+1}du = 2\int \frac{s^2}{s^4+1}ds$$

Next week: a clever trick.

S11.1 – Differential Equations – Modeling the World

Assaf Bar-Natan

"You realize that life goes fast
It's hard to make the good things last
You realize the sun doesn't go down
It's just an illusion caused by the world spinning round"

–"Do You Realize??", The Flaming Lips

Feb. 5, 2020

What Is a Differential Equation?

A differential equation is an algebraic relation between functions and their derivatives. For example:

$$f'(t) = 4$$

$$f''(t) = f'(t) + 1$$

$$F = ma = m\frac{d^2 s}{dt^2}$$

Sometimes, these differential equations have solutions.

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Sometimes, these differential equations have solutions.
For which values of k is $te^t - e^t + k$ a solution to the differential equation:

$$f'(t) = f(t) + e^t$$

Key Points from Reading

In groups of $3-4$, take turns listing a key point from the reading. Make sure to explain why you think these are key points.

Submissions Closed

Sort the following key points and ideas from the reading in decreasing importance

✓ **2%** Answered Correctly

Correct Order

- **B** Setting up an algebraic model of differential equations
- **C** Estimating solutions to differential equations numerically
- **D** Using initial conditions we can find constant terms in solutions
- **A** General solutions vs particular solutions
- **E** To solve a differential equation we rearrange and integrate

February 4 at 10:59 PM results

Condense Text

181/181 answered

Ask Again

Open | Closed | Responses | Correct

100%

The SI Model – Cat's Cold

The cats are sick with a cold. For now, we will make the following assumptions:

- A cat is either susceptible to the virus or infected
- A cat that is infected stays infected
- Let $S(t)$ be the number of susceptible cats after t days
- Let $I(t)$ be the number of infected cats after t days

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- Let $I(t)$ be the number of infected cats after t days

Question: Explain why $\frac{dI}{dt} = -\frac{dS}{dt}$

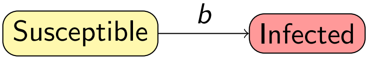
A model for infection

1:00 Show Correct Answer

Suppose that each infected cat licks $\frac{1}{c}$ of the susceptible cats in a day, and that $\frac{1}{a}$ of licks result in a new infection. Given the number of cats infected at time t, what is a good estimate for the number of cats infected at time t+1?

All results

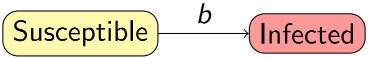
A	$I(t+1) = I(t) + \frac{1}{a}\frac{1}{c}$	18
B	$I(t+1) = \frac{1}{a}I(t) + \frac{1}{c}S(t)$	34
C	$I(t+1) = \frac{1}{a}\frac{1}{c}I(t)S(t) + I(t)$	119
D	$I(t+1) = \frac{1}{a}\frac{1}{c}S(t)I(t)$	13



If we define $b = \frac{1}{ac}$, then we know that:

$$I(t+1) - I(t) = bI(t)S(t)$$

Deriving the SI Model – Cat's Cold



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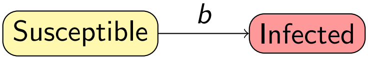


If we define $b = \frac{1}{ac}$, then we know that:

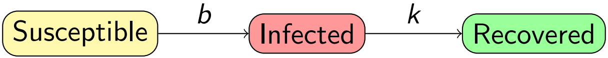
$$I(t+1) - I(t) = bI(t)S(t)$$

Question: What is the verbal interpretation of $I'(t)$?
$I'(t) = A$ means that A new cats have been infected between t and $t+1$. In other words, $I(t+1) - I(t) = A$. So we can write:

$$I'(t) = bI(t)S(t)$$

SIR Model – The Cats' Recovery

Eventually, all the cats are infected. Luckily, they start recovering.



Every day, a fraction $k < 1$ of the infected cats end up recovering.
Let $R(t)$ be the number of cats recovered at day t.
Question: Write an expression for $R(t+1) - R(t)$

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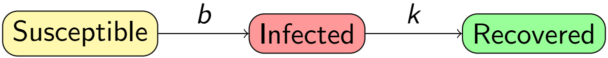

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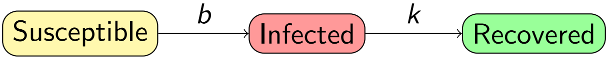

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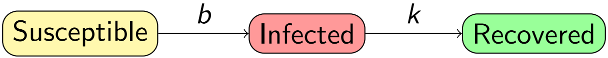

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$$R'(t) = kI(t)$$

SIR Model – The Cats' Recovery

We now add the following assumptions:

- Cats can be susceptible, infected, or recovered.
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- An infected cat can recover from the virus.

When all of the cats have been infected, and start recovering, we have $I + R = constant$, so

$$\frac{dI}{dt} = -\frac{dR}{dt} = -kI(t)$$

Assume that at $t = 0$, all 30 cats were infected, and none have recovered. If $k = 0.4$, how many cats will have recovered after 3 days? You might want to use the table below as a guide:

t	0	1	2	3
$I(t)$	30			
$I'(t) \approx$				

Submissions Closed

The equation $I(t) = 30e^{-0.4t}$ is a general solution to the differential equation

$$\frac{dI}{dt} = -0.4I(t)$$

✓ **68%** Answered Correctly

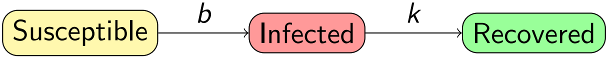
A	True	50
B	False	122
C	This is not a solution to the differential equation	8

February 4 at 11:12 PM results | Segment Results | Compare with session | Show percentages | Hide Graph | Condense Text

180/183 answered | Ask Again

Open | Closed | Responses | Correct | 100%

The SIR Model – Wrap-Up



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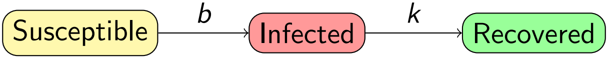

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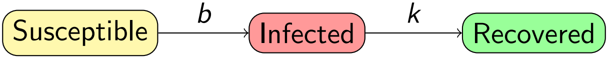

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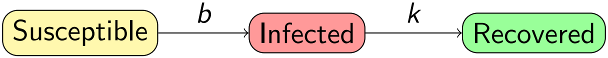
$$
\begin{aligned}
S'(t) &= -bI(t)S(t) \\
I'(t) &= bI(t)S(t) - kI(t) \\
R'(t) &= kI(t)
\end{aligned}
$$

The SIR Model – Wrap-Up



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$$S'(t) = -bI(t)S(t)$$
$$I'(t) = bI(t)S(t) - kI(t)$$
$$R'(t) = kI(t)$$

What are some of the shortcomings of the SIR model?

Plans for the Future

For next time:

Review session

For Monday:

WeBWork 11.2 and actively read section 11.2