Welcome to MAT135 LEC0501 (Assaf)

Think of a hobby or a skill you have. Did you get a chance to do it this year?

S7.7 – Improper Integrals – Comparisons, Estimation, and Guessing

Assaf Bar-Natan

"Laughing like children, living like lovers
Rolling like thunder under the covers
And I guess that's why they call it the blues"

–"I Guess That's Why They Call it the Blues", Elton John

Feb. 3, 2020

The CIQ

Things I noticed

- TopHat and working together got a lot of people engaged
- Things we dislike:
  - When people aren't participating
  - **When Assaf skips things**
  - **Unexplained answers**
- Things we like:
  - Explaining after TopHats
  - Other people helping us understand
- **Reading summary at the start of class before self-work**
- Things that surprised you:
  - How welcoming you were to each other
  - How many friends you made
  - The style of the class

How Do We Learn?

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What is something you are good at? How did you learn it?

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Why do we ask you to read before class?

Good Reading Strategies

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Three-time rule:

- **Skim** – don't worry about understanding, just read! (10 mins)
- **Note** – take meticulous notes, and read carefully! (one hour)
- **Own** – Read things one last time to pick up pieces you've missed (10 mins)

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Other ideas:

- Ask friends for help
- **TAKE NOTES**
- Do the problems

Ice-Cream Sandwich

Spend a minute to think about:

- Something in the chapter that you've mastered.
- Something in the chapter that you've learned
- Something in the chapter that you've got to revisit

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Share with your neighbours

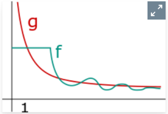
The Idea of Comparisons

"If it looks like a cat, and meows like a cat, it converges like a cat"

If $f \leq g$ then $\int_a^b f \leq \int_a^b g$, so if g converges, then f converges.

Submissions Closed

Assume that $\int_1^\infty g(x)\,dx$ converges. What can you say about $\int_1^\infty f(x)\,dx$?



✓ **69%** Answered Correctly

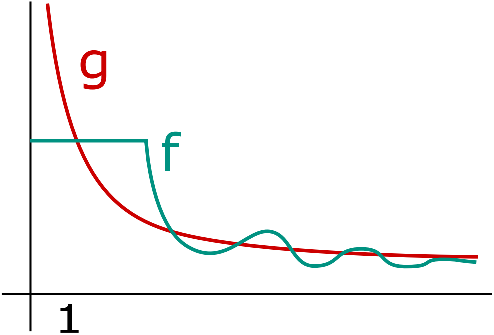
A	It converges	118
B	It diverges	19
C	We can't tell anything from this picture	33

February 3 at 12:31 AM results | Segment Results | Compare with session | **Show percentages** | **Hide Graph** | **Condense Text**

170/170 answered | Ask Again

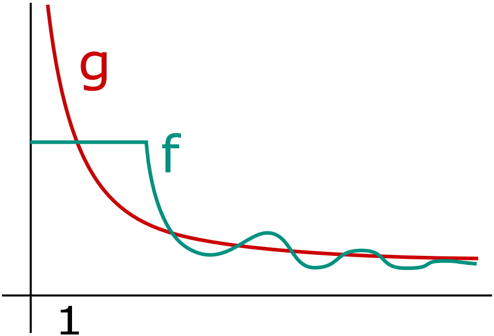
Open | Closed | Responses | Correct | 100%

A Graphical Example



If $\int_1^\infty g(x)dx$ converges, what can we say about $\int_1^\infty f(x)dx$?
It must converge, by the comparison test, since f looks like g.
If $\int_0^1 g(x)dx$ diverges, what can we say about $\int_0^1 f(x)dx$?

A Graphical Example



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It must converge, by the comparison test, since f looks like g.
If $\int_0^1 g(x)dx$ diverges, what can we say about $\int_0^1 f(x)dx$?
$\int_0^1 f(x)dx$ is not an improper integral, so it converges.

Takeaway

When looking at what integrals to infinity do, we only care about the tail. If the tails look similar, then the functions converge and diverge together.

Spot the Error

Peek, the curious cat, is trying to compute:

$$\int_1^\infty \frac{-1}{x} dx$$

She writes:

"I know that

$$\int_1^\infty \frac{1}{x^2} = 1$$

I also know that $\frac{-1}{x} \leq \frac{1}{x^2}$ for all $x \geq 1$
So by the comparison test, I can conclude that $\int_1^\infty \frac{-1}{x} dx$ converges."

What was her mistake? Write a takeaway from this example.

Takeaway

When dealing with negative integrands, we can't just bound things from one side.

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Aside: This should remind some of you of the squeeze theorem...

Review: Known Improper Integrals

Integral	Condition on parameter (p or a)	Converges/diverges
$\int_0^1 x^p$	$p > -1$	
$\int_0^1 x^p$	$p \leq -1$	
$\int_1^\infty x^p$	$p \geq -1$	
$\int_1^\infty x^p$	$p < -1$	
$\int_0^\infty e^{-ax}$	$a > 0$	
$\int_0^\infty e^{-ax}$	$a \leq 0$	

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$\int_1^\infty x^p$	$p < -1$	Converges
$\int_0^\infty e^{-ax}$	$a > 0$	Converges
$\int_0^\infty e^{-ax}$	$a \leq 0$	Diverges

An Algebraic Example

"If it looks like a cat, and meows like a cat, it converges like a cat"

What known improper integrals do the following integrals look like:

$$\int_6^\infty \frac{1}{(x-5)^2}dx$$

$$\int_0^5 \frac{1+\sin^2(x)}{\sqrt{x}}dx$$

$$\int_5^\infty \frac{1+\sin^2(x)}{\log(x)}dx$$

Meows Like a Cat

$$\int_6^\infty \frac{1}{(x-5)^2}\,dx$$

Key ideas:

- $x-5<x$ so $\frac{1}{(x-5)^2} \geq \frac{1}{x^2}$. This won't help.
- Substitute $u=x-5$ to get $\int_1^\infty \frac{1}{u^2}du$
- When x is big, $\frac{1}{(x-5)^2} \approx \frac{1}{x^2}$

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This integral converges

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$$\int_0^5 \frac{1+\sin^2(x)}{\sqrt{x}}\,dx$$

Key ideas:

- $\frac{1}{\sqrt{x}} \leq \frac{1+\sin^2(x)}{\sqrt{x}} \leq \frac{2}{\sqrt{x}}$
- When x is small, integrand looks like $\frac{1}{\sqrt{x}}$

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$$\int_5^\infty \frac{1+\sin^2(x)}{\log(x)}\,dx$$

Key ideas:

- The $1+\sin^2(x)$ term is a distraction that just oscillates a bit.
- Looks like $\int_5^\infty \frac{1}{\log(x)}$
- When x is big, $x > \log(x)$ so $\frac{1}{x} < \frac{1}{\log(x)}$

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- Looks like $\int_5^\infty \frac{1}{\log(x)}$
- When x is big, $x > \log(x)$ so $\frac{1}{x} < \frac{1}{\log(x)}$

This integral diverges

Takeaway

When comparing integrals, be mindful of easy substitutions, but also watch for the bounds!

The Cat's Tail

Does the integral:

$$\int_a^\infty \frac{1}{x^2}\,dx$$

(where $a > 1$) converge?

The Cat's Tail

Does the integral:

$$\int_a^\infty \frac{1}{x^2}\,dx$$

(where $a > 1$) converge? Yes!

$$\int_1^\infty \frac{1}{x^2}\,dx = \int_1^a \frac{1}{x^2}\,dx + \int_a^\infty \frac{1}{x^2}\,dx$$

So we get:

$$1 = 1 - \frac{1}{a} + \int_a^\infty \frac{1}{x^2}\,dx$$

and we can solve for the integral.

Plans for the Future

For next time:

WeBWork 11.1 and actively read section 11.1