Welcome to MAT135 LEC0501 (Assaf)

The Borwein integrals:

$$\int_0^\infty \frac{\sin(x)}{x}\,dx = \frac{\pi}{2}$$

$$\int_0^\infty \frac{\sin(x)}{x}\frac{\sin(x/3)}{x/3}\,dx = \frac{\pi}{2}$$

$$\int_0^\infty \frac{\sin(x)}{x}\frac{\sin(x/3)}{x/3}\frac{\sin(x/5)}{x/5}\,dx = \frac{\pi}{2}$$

$$\int_0^\infty \frac{\sin(x)}{x}\frac{\sin(x/3)}{x/3}\cdots\frac{\sin(x/13)}{x/13}\,dx = \frac{\pi}{2}$$

$$\int_0^\infty \frac{\sin(x)}{x}\frac{\sin(x/3)}{x/3}\cdots\frac{\sin(x/15)}{x/15}\,dx = \frac{467807924713440738696537864469}{935615849440640907310521750000}\pi \approx \frac{\pi}{2} - 2.31\times 10^{-11}$$

Improper Integrals – Going to Infinity

Assaf Bar-Natan

"Out all night, sun's too bright
Though I'm blind, it'll be all right
Going to infinity
What does it mean?
Infinity"

–"What Does it Mean?", The Flaming Lips

Jan. 30, 2020

Reading Comprehension – Fill in Blanks

An integral $\int_a^b f(t)dt$ is an improper intergral when ______ are infinite or when the ______ is infinite.

Reading Comprehension – Fill in Blanks

The faster $f(t)$ decreases as _____, the more likely that $\int_a^\infty f(t)dt$ __________

Reading Comprehension – Fill in Blanks

An improper integral is defined as a _____ of definite integrals.

Reading Comprehension – Fill in Blanks

Suppose that $\lim_{x\to b} f(x) = \infty$**. If** $\lim_{x\to b} \int_a^x f(t)dt$ ______, **we define** $\int_a^b f(t)dt$ **by** ____________. **Otherwise, we say that** $\int_a^b f(t)dt$ ______.

Reading Comprehension – Fill in Blanks

If $\lim_{x\to\infty} \int_a^x f(t)dt$ ______**, we define** $\int_a^\infty f(t)dt$ **by** ______**, and we say that** $\int_a^\infty f(t)dt$ ______**.**

Submissions Closed

Click on the first statement in the following argument that is incorrect

✓ 14% Answered Correctly

Marzipan is trying to compute the integral $\int_{-6}^{6} \frac{1}{x} dx$. She writes:

$$\int_{-6}^{6} \frac{1}{x} dx = [\log(|x|)]_{-6}^{6}$$

$$= \log(|6|) - \log(|-6|) = 0$$

Thus, the integral $\int_{-6}^{6} \frac{1}{x} dx$ converges and is equal to 0.

Invalid date

192/192 answered

Ask Again

Open | Closed | Responses | ✓ Correct

100%

Takeaway

The fundamental theorem only works when the integrand is continuous. If f is infinite between the bounds, the integral is improper!

What is an Improper Integral?

Worth **1 participation point**and **0 correctness points**

ⓘ **Multiple answers:** Multiple answers are accepted for this question

Which of the following are improper integrals? (select all)

All results ▾

A	$\int_a^\infty \frac{\sin(x)}{x}dx$	166
B	$\int_4^5 \frac{1}{x}dx$	10
C	$\int_0^1 \frac{1}{2-3x}dx$	139
D	$\int_1^2 \log(x)dx$	33
E	$\int_1^2 \frac{1}{2x-1}dx$	33

An Example

We will determine if $\int_{-6}^{6} \frac{1}{x} dx$ converges.

An Example

We will determine if $\int_{-6}^{6} \frac{1}{x} dx$ converges.

Write a list of steps you should take to determine this.

An Example

We will determine if $\int_{-6}^{6} \frac{1}{x} dx$ converges.

Write a list of steps you should take to determine this.

- Split the integral into two improper integrals
- Turn each integral into a limit
- Take the limit
- Do the limits converge?

An Example – Splitting the Integral

- **Split the integral into two improper integrals**
- Turn each integral into a limit
- Take the limit
- Do the limits converge?

An Example – Splitting the Integral

- **Split the integral into two improper integrals**
- Turn each integral into a limit
- Take the limit
- Do the limits converge?

The function $f(x) = \frac{1}{x}$ goes to ∞ when $x \to 0$, so we should split the integrals there.

$$\int_{-6}^{6} \frac{1}{x} dx = \int_{-6}^{0} \frac{1}{x} dx + \int_{0}^{6} \frac{1}{x} dx$$

Now, we should solve each of these as an improper integral.

An Example – Turning it Into a Limit

- Split the integral into two improper integrals
- **Turn each integral into a limit**
- Take the limit
- Do the limits converge?

$$\int_{-6}^{6} \frac{1}{x} dx = \int_{-6}^{0} \frac{1}{x} dx + \int_{0}^{6} \frac{1}{x} dx$$

An Example – Turning it Into a Limit

- Split the integral into two improper integrals
- **Turn each integral into a limit**
- Take the limit
- Do the limits converge?

$$\int_{-6}^{6} \frac{1}{x} dx = \int_{-6}^{0} \frac{1}{x} dx + \int_{0}^{6} \frac{1}{x} dx$$

We need to check if the following limits exist:

$$\lim_{b \to 0^-} \int_{-6}^{b} \frac{1}{x} dx$$

$$\lim_{a \to 0^+} \int_{a}^{6} \frac{1}{x} dx$$

An Example – Taking the Limit

- Split the integral into two improper integrals
- Turn each integral into a limit
- **Take the limit**
- Do the limits converge?

We need to check if the following limits exist:

$$\lim_{b \to 0^-} \int_{-6}^{b} \frac{1}{x}\,dx$$

$$\lim_{a \to 0^+} \int_{a}^{6} \frac{1}{x}\,dx$$

An Example – Taking the Limit

- Split the integral into two improper integrals
- Turn each integral into a limit
- **Take the limit**
- Do the limits converge?

We need to check if the following limits exist:

$$\lim_{b \to 0^-} \int_{-6}^{b} \frac{1}{x} dx$$

$$\lim_{a \to 0^+} \int_{a}^{6} \frac{1}{x} dx$$

We compute:

$$\lim_{b \to 0^-} \int_{-6}^{b} \frac{1}{x} dx = \lim_{b \to 0^-} (\log(b) - \log(|-6|)) = -\infty$$

$$\lim_{a \to 0^+} \int_{a}^{6} \frac{1}{x} dx = \lim_{a \to 0^+} (\log(6) - \log(|a|)) = \infty$$

An Example – Taking the Limit

- Split the integral into two improper integrals
- Turn each integral into a limit
- Take the limit
- **Do the limits converge?**

What does this tell us about $\int_{-6}^{6} \frac{1}{x} dx$? Plug this in to WolframAlpha!

Takeaway

When evaluating improper integrals, you might need to split them up!

Submissions Closed

If $\lim_{x \to \infty} f(x) = 0$ then $\int_1^{\infty} f(x)\,dx$ converges

✓ 38% Answered Correctly

A	True, and I can prove it	39
B	True, but I'm not sure	78
C	False, but I'm not sure	43
D	False, and I have a counter-example	30

January 30 at 11:47 PM results   Segment Results   Compare with session   Show percentages   Hide Graph   Condense Text

190/190 answered   Ask Again

Open   Closed   Responses   ✓ Correct   100%

Criterion For Convergence

For which p does $\int_0^1 x^p dx$ converge?

Criterion For Convergence

For which p does $\int_0^1 x^p dx$ converge?

- Get into groups, and assign each group member a different number.

Criterion For Convergence

For which p does $\int_0^1 x^p dx$ converge?

- Get into groups, and assign each group member a different number.

2 min Check whether $\int_0^1 x^p dx$ converges where p is your number.

Criterion For Convergence

For which p does $\int_0^1 x^p dx$ converge?

- Get into groups, and assign each group member a different number.

2 min Check whether $\int_0^1 x^p dx$ converges where p is your number.

2 min Compare answers with your neighbours, and form a conjecture.

Criterion For Convergence

For which p does $\int_0^1 x^p dx$ converge?

- Get into groups, and assign each group member a different number.

2 min Check whether $\int_0^1 x^p dx$ converges where p is your number.

2 min Compare answers with your neighbours, and form a conjecture.

2 min Check your conjecture by hand or on WolframAlpha

Takeaway

The integral $\int_0^1 x^p dx$ **converges when** $p > -1$

Roy and the Big Barn

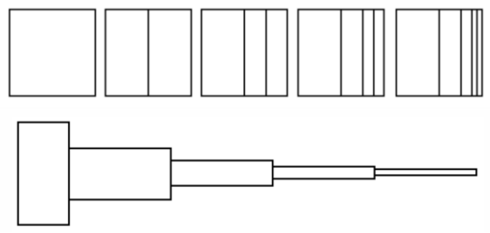
Roy the kitten is walking around the barn, and says the following:

> "I know this barn in and out, and I can confidently say that it has a finite area. I don't know its shape, but because it has finite area, I should be able to circumnavigate it in finite time."

Write a sentence explaining to Roy where he is wrong. Be sure to give an example.

Roy and the Big Barn

Here's a helpful picture:



Plans for the Future

For next time:
WeBWork 7.7 and read section 7.7