Welcome to MAT135 LEC0501 (Assaf)

What is the integral $\int \frac{1}{\text{cabin}} d\text{cabin}$?

Two challenging integrals from last week:

$$\int \sin(e^t)dt = Si(e^x) + C$$

For $\int \sqrt{\tan(x)}$, substitute $u = \tan(x)$ to get:

$$\int \frac{\sqrt{u}}{u^2+1} = ???$$

This is very hard. Further developments next week.

S7.2 – Integration Methods – Integration by Parts

Assaf Bar-Natan

"Sometimes I lie awake, night after night
Coming apart at the seams
Eager to please, ready to fight
Why do I go to extremes?"

–"Why Do I Go To Extremes", Billy Joel

Jan. 27, 2020

Reading Comprehension

- The differentiation rule that gives us integration by parts is the ______ rule.
- The integration by parts technique tells us that $\int uv' dx = \_\_\_\_\_\_ - \_\_\_\_\_\_.$

Submissions Closed

True / False: When we use integration by parts to compute definite integrals, we need to change the limits of the definite integral.

✓ **57%** Answered Correctly

A	True	26
B	False	99
C	Only in some cases.	49

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174/174 answered · Ask Again

Open · Closed · Responses · ✓ Correct · 100%

Takeaway

When faced with an integral that is a product of functions, try integration by parts. (Sometimes this will work for other functions too)

Leibniz and The Product Rule

MANUSCRIPT DATED NOV. 21, 1675. 107

Let us seek to obtain others in addition, such as

$$\int t\,dy = \int y\,dx.$$

Now this furnishes us with nothing new; but $\int tw + \int xw = xy$ or $t\,dy + x\,dy = \overline{dxy}$, and $t = \dfrac{dx}{dy}y$; hence the latter $= \dfrac{\overline{dxy} - x\,dy}{dy}$. Therefore $\overline{dx}\,y = \overline{dxy} - x\,\overline{dy}$.

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Leibniz used the fundamental theorem and integration by parts to conclude the product rule!

Building Fur – I.B.P Example

A cold snap hits the cats, and Mia's body starts building up her fur at a rate of $f(t)$ pounds per day. If $f(t) = 0.5 * t^2 e^{-t}$, how much hair has she built up after ten days?

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- Does this simplify the question?

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$$\begin{aligned} u &= t^2 & v' &= e^{-t} \\ u' &= 2t & v &= -e^{-t} \end{aligned}$$

gives:

$$\int_0^{10} 0.5t^2 e^{-t} dt = \left[0.5t^2(-e^{-t})\right]_0^{10} + \int_0^{10} te^{-t} dt$$

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Does this simplify the question enough to solve? Evaluating the last integral, and plugging in the bounds gives $\int_0^{10} 0.5t^2 e^{-t} dt \approx 0.997$.

Submissions Closed

Match the integral to the first technique you would use to compute it.

✓ **63%** Answered Correctly

Correct Order

	Integral			Technique	Responses
1	$\int e^{2x}\sin(e^x)\,dx$	→	B	substitution	149
2	$\int e^{2x}\cos x\,dx$	→	D	integration by parts	142
3	$\int x^3\cos(x)\,dx$	→	D	integration by parts	146
4	$\int x(2+x^2)\ln(2+x^2)\,dx$	→	B	substitution	139

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Condense Text

186/186 answered

Ask Again

Open | Closed | Responses | Correct

100%

Takeaway

Integration by parts is useful when there is a product of functions, and we want one of them to "disappear".

dETAILS Mnemonic

$$\int uv'dx = uv - \int vu'dx$$

Here is a mnemonic for what functions to use for v' (read backwards for what functions to use as u)

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- **A** lgebraic (ie polynomials, ratios of polynomials)
- **I** nverse trigonometric
- **L** ogarithms
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Bonus: find $\int xSi(x)dx$.

Integration by Parts – Functions Given Strangely

Let's say we have two functions, f, and g. g is given as a table of values, and f is given as a formula. Which of the following are easy to estimate?

- $f \cdot g$ evaluated at some point
- $\int_a^b f' g dx$
- $\int_a^b f g' dx$
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Hint: For which of these integrands can you write a table of values?

Integration by Parts – Functions Given Strangely

Let's say we have two functions, f, and g. g is given as a table of values, and f is given as a formula.

$$\int_a^b f(x)g'(x) = [fg]_a^b - \int_a^b f'(x)g(x)dx$$

We can now write a table for $f'(x)$, for $g(x)$, and $f'(x)g(x)$, and estimate the integral on the right.

What is Easy to Compute?

Worth **1 participation point**and **0 correctness points**

What is Easy to Compute?

Show Correct Answer

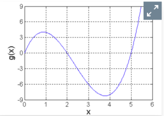
Let's say that f(x) is a function given as a formula, and g(x) is a function given as a table of values. Which of the following can you easily estimate?

All results

A	$f(a)g(a)$ for some value of a	86
B	$\int_a^b f(x)g(x)dx$	16
C	$\int_a^b f'(x)g(x)dx$	38
D	$\int_a^b f(x)g'(x)dx$	39
E	$\int_a^b f'(x)g'(x)dx$	1

Submissions Closed

Estimate $\int_0^5 f(x)g'(x)\,dx$ if $f(x) = 2x$ and $g(x)$ is given by the graph below.



✓ 62% Answered Correctly

A	40	23
B	20	121
C	10	30
D	-10	21
E	This integral cannot be done using integration by parts	0

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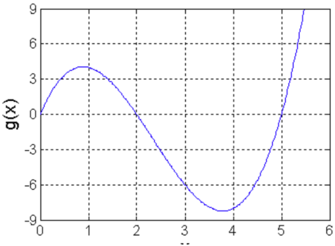
Graphical Estimation



$$\int_0^5 f(x)g'(x) = f(5)g(5) - f(0)g(0) - \int_0^5 g(x)f'(x)dx$$

$$= -\int_0^5 2g(x)$$

Spot The Error

The Calculus Cats find a note on the floor. It reads:

To compute $\int \tan(x)dx$, we integrate by parts.

$$u = \frac{1}{\cos(x)} \qquad v' = \sin(x)$$

$$u' = \tan(x)\sec(x) \qquad v = -\cos(x)$$

so

$$\int \tan(x)dx = \int uv'dx = uv - \int vu'dx = -1 + \int \tan(x)$$

Simplifying, we get $0 = -1$.

The cats are stressed by this, to say the least. Can you help them?

Spot The Error

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To compute $\int_{\pi/6}^{\pi/4} \tan(x) dx$, we integrate by parts.

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so

$$\int_{\pi/6}^{\pi/4} \tan(x) dx = \int_{\pi/6}^{\pi/4} uv' dx = uv - \int_{\pi/6}^{\pi/4} vu' dx = -1 + \int_{\pi/6}^{\pi/4} \tan(x)$$

Simplifying, we get $0 = -1$.

The cats are even more stressed by this. Can you help them?

Plans for the Future

For next time:

Integration and Computer Algebra Systems. No reading, but bring a computer, and review what we've done!