MAT 309, Fall 2019
Problem Set 2, submit on Crowdmark by the start of class on Thursday October 3

Problem 1: Section 2.4, Exercise 5 (page 54). For each of (a)–(c), decide whether or not φ is a propositional consequence of Γ and justify your assertion.

Note: In part (a), assume $\exists x \neg R(x)$ is an abbreviation for the formula $\neg \forall x R(x)$. (Is the answer different if $\exists x \neg R(x)$ is assumed to abbreviate $\neg \forall x \neg\neg R(x)$?)

Problem 2. Prove that the logical axiom (E3) is valid.

Problem 3: §2.8 Exercise 2 (page 70). For each of structures (a)–(e), decide whether or not it satisfies the axioms for a *dense linear order without endpoints.* If not, point out which axiom the structure fails to satisfy.

Problem 4. This problem will give you experience finding deductions. (For concrete examples of deductions, see §2.7 Exercises 5–7 on page 66 and their solutions beginning on page 296.) Let $\mathcal{L}$ be an arbitrary language.

(a) Show that $\vdash t = t$ for all terms t.

Hint: Use logical axioms (E1) and (Q1). You may also use the fact (proved in class) that $\vdash \alpha$ if and only if $\vdash \forall x \alpha$, for any formula α and variable x.

(b) Assume f is a 1-ary function symbol in $\mathcal{L}$. Give a deduction showing

$$\vdash (\forall x)[\,(\exists y)(fx = y)\,].$$

(c) Show that

$$\vdash (\forall x)[\,(\forall y)(fx = fy)\,] \rightarrow (\exists z)[\,(\forall y)(z = fy)\,].$$

(In English, this formula states: "If f is a function such that $f(x) = f(y)$ for all x and y, then there exists some z such that $z = f(y)$ for all y.")

Hint: Start by using the Deduction Theorem (Theorem 2.7.4 in the book).