MAT 309, Fall 2019
Problem Set 1, due at the start of class on Thursday September 19

Problem 1.

(a) Rewrite the following terms in Polish (prefix) notation without using parentheses (that is, as strings according to formal syntax of first-order logic):

$$(x + y) + z, \qquad x + (y + z), \qquad (x + (y \cdot z)) \cdot ((y \cdot x) + z).$$

(b) §1.5, Exercise 1 (page 21): Identify the free variables.

(c) §1.8, Exercise 1 (page 36): Write out u_t^x.

(d) §1.8, Exercise 2 (page 36): Write out ϕ_t^x and decide if t is substitutable for x in ϕ.

(e) Let $A, B, C \in \{T, F\}$ be propositional variables. Verify that

$$P :\equiv (\neg(A \wedge B) \vee C) \leftrightarrow (A \to (B \to C))$$

is a propositional tautology by writing out a truth table:

A	B	C	$\neg(A \wedge B)$	$\neg(A \wedge B) \vee C$	$B \to C$	$A \to (B \to C)$	P
T	T	T					
T	T	F					
T	F	T					
T	F	F					
F	T	T					
F	T	F					
F	F	T					
F	F	F					

Problem 2.

(a) §1.4, Exercise 6 (page 18): Prove that if s and t are distinct terms of a language $\mathcal{L}$, then s is not an initial segment of t. Argue by induction on the length of s (i.e., the number of symbols in s).

(b) §1.4, Exercise 7: Prove the *unique readability property* for terms.

Problem 3. In this problem, let $\mathcal{L}$ be the language $\{0, S, +\}$ where 0 is a constant symbol, S is a unary function symbol, and $+$ is a binary function symbol.

(a) Let $\mathbb{N}$ be the $\mathcal{L}$-structure with universe $\{0, 1, 2, \dots\}$ (the natural numbers) where $0^{\mathbb{N}}$, $S^{\mathbb{N}}$ and $+^{\mathbb{N}}$ are the usual zero, successor function and addition. Write down a variable-free term t, other than $SS0$, such that $\mathbb{N} \models (t = SS0)$.

(b) Let $\mathbb{Z}$ be the $\mathcal{L}$-structure with universe $\{\dots, -2, -1, 0, 1, 2, \dots\}$ (the integers) where $0^{\mathbb{Z}}$, $S^{\mathbb{Z}}$ and $+^{\mathbb{Z}}$ are the usual zero, successor function and addition. Write down two different $\mathcal{L}$-sentences ϕ such that $\mathbb{Z} \models \phi$ and $\mathbb{N} \models \neg\phi$. In the first sentence ϕ_1, avoid using the symbol $+$ (you may use 0 and S). In the second sentence ϕ_2, avoid using S (you may use 0 and $+$).

(c) Define an $\mathcal{L}$-structure $\mathfrak{A}$ such that $\mathfrak{A} \models \neg(t_1 = t_2)$ for every two distinct variable-free terms t_1 and t_2.