CSC 463 - Winter 2019
Problem Set 2, due at the start of tutorial on Friday February 15

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(1) Show that a language B is semi-decidable if, and only if, $B \leq_m A_{TM}$.

(2) Let $R = \{\langle M \rangle \mid M$ is a Turing machine that accepts w^{reverse} whenever it accepts $w\}$. Without using Rice's Theorem, show that R is undecidable.

(3) Let $\Gamma = \{0, 1, \sqcup\}$ be the tape alphabet for all Turing machines in this problem. Define the **busy beaver function** $BB : \mathbb{N} \to \mathbb{N}$ as follows. For each value of k, consider all k-state Turing machines that halt when started with a blank tape. Let $BB(k)$ be the maximum number of 1s that remain on the tape among all of these machines. Show that BB is not computable.

Note: We say that a function $\mathbb{N} \to \mathbb{N}$ is *computable* iff the corresponding function $\{0,1\}^* \to \{0,1\}^*$ (on binary representations of natural numbers) is computable.

(4) Show that for every language A, there exists a language B such that $B \not\leq_m A$. (Hint: Argue about countable vs. uncountable sets.)

Next, show that there exists a language C such that $A \leq_m C$ and $C \not\leq_m A$.

(5) Extra credit (worth up to 10% of the assignment):

Let $A \subseteq \{0,1\}^*$ be semi-decidable. For $n \in \mathbb{N}$, let $A_n = A \cap \{0,1\}^n$. Show that there is a constant c such that for all $n \in \mathbb{N}$ and $x \in A_n$,

$$K(x) \leq \log |A_n| + 2 \log n + c.$$