

Quiz 2

MAT475H1F Fall 2026

Name: _____

Student number: _____

Grade: _____ / 5

Question.

[5 marks]

A real number is written in each square of a 4×4 grid. The number in each square is the average of the numbers in the squares that share an edge with it. Thus a corner square has two neighbors, a non-corner boundary square has three, and an interior square has four.

Prove that all sixteen numbers are equal.

Solution. Consider the squares whose value differs from that of at least one neighbour. If there are no such squares, all neighbouring squares have the same value. Since the grid is connected, all sixteen values are equal, and we are done.

Otherwise, among these squares choose one with the largest value, say M . No neighbour can have a value greater than M : that neighbour would itself differ from a neighbour (our chosen square), contradicting our maximal choice. Thus every neighbour has value at most M . By our choice of square, at least one neighbour has a different value, so its value is strictly less than M .

The average of the neighbouring values is therefore strictly less than M , contradicting the hypothesis. This contradiction completes the proof. $\square$