Quiz 0

MAT475H1F Fall 2026

Name: ____________________

Student number: ____________________

Grade: ________ / 5

Question. **[5 marks]**

Please read the claim below and its proof carefully. Do you believe the claim? Give a detailed explanation of your answer. Simply saying "the induction is wrong" is not enough; explain why the argument fails.

Claim. All horses have the same colour.

Proof. For each integer $n \geq 1$, let $P(n)$ be the statement that any group of n horses consists entirely of horses of the same colour. We prove $P(n)$ by induction.

The base case $P(1)$ is clear: a single horse has the same colour as itself.

Now assume that $P(n)$ is true, and consider any group of $n+1$ horses, numbered $1, 2, \ldots, n+1$. By the induction hypothesis, horses $1, 2, \ldots, n$ all have the same colour. Also by the induction hypothesis, horses $2, 3, \ldots, n+1$ all have the same colour. These two groups overlap, so their common horses show that both groups have the same colour. Therefore all $n+1$ horses have the same colour.

By induction, $P(n)$ holds for every $n \geq 1$. $\square$

Solution. The claim is false.

The error in the proof occurs in the induction step when $n = 1$. In that case, the group of $n + 1 = 2$ horses consists of horses 1 and 2. The first group of n horses is $\{1\}$ and the second is $\{2\}$. So the two groups do not overlap and there is no common horse whose colour connects the two groups. The argument does not prove $P(2)$ from $P(1)$.

When $n \geq 2$, the two groups share horses $2, \ldots, n$, so that part of the induction step works. That is, $P(2)$ does imply $P(n)$, for $n \geq 2$. But the proof cannot get from its base case to $P(2)$ and the induction never gets started.

No horses were harmed in the making of this quiz.