# MAT257 Tutorial Worksheet 8

Adriano Pacifico – Optimize your studying

**Problem 1.** Find the extrema of $f(x, y, z) = x^2 + 2y^2 + 3z^2$ on the unit sphere $x^2 + y^2 + z^2 = 1$.

**Problem 2.** For each of the following functions, find its critical points and determine the nature of the critical point (e.g. local min, local max, saddle or degenerate critical point).

(a) $y(x) = x^{2019}$

(b) $f(x, y) = x^4 - 2x^2 + y^3 - 6y$

(c) $g(x, y) = x^2 + 3y^4 + 4y^3 - 12y^2$

(d) $h(x, y) = (x - 1)(x^2 - y^2)$

**Problem 3.** Consider the quadratic function

$$f(x, y) = ax^2 + bxy + cy^2.$$

Under what conditions (this means find necessary and sufficient conditions) on $a, b, c$ does $f$ have a local maximum at $(0, 0)$? A local minimum? A saddle point?

**Problem 4.** Find the largest volume of a box inscribed in the unit sphere.

**Problem 5.** Show the function

$$f(x, y) = x^4 + 6x^2y^2 + y^4 - \frac{9}{4}x - \frac{7}{4}y$$

achieves a global minimum on $\mathbb{R}^2$ and find the value of this minimum.

**Problem 6. (Hard)** Let $A$ be a real symmetric $n \times n$–matrix and define $f : \mathbb{R}^n \to \mathbb{R}$ by $f(x) = Ax \cdot x$. Show the maximum and minimum of $f$ on the unit sphere, $S^{n-1} = \{x \in \mathbb{R}^n \ : \ ||x|| = 1\}$, is the largest and smallest eigenvalues of $A$ respectively.