MAT257 Tutorial Worksheet 6

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Problem 1.

(a) Show there is no continuous map $f : \mathbb{R}^3 \to \mathbb{R}^2$ such that $f(S^2) = \mathbb{R}^2$, where $S^2 = \{(x, y, z) \in \mathbb{R}^3 \ : \ x^2 + y^2 + z^2 = 1\}$ denotes the unit sphere.

(b) Does there exist a continuous map $f : \mathbb{R}^2 \to \mathbb{R}^3$ with $f(\mathbb{R}^2) = S^2$?

Problem 2. Let M be a subset of $\mathbb{R}^n$. Suppose $f : M \to \mathbb{R}$ is a function whose graph,

$$\Gamma_f = \{(x, y) \in \mathbb{R}^n \times \mathbb{R} \ : \ x \in M, \ y = f(x)\},$$

is compact. Show that f is continuous.

Problem 3. Let $f, g : \mathbb{R}^2 \to \mathbb{R}$ and $h : \mathbb{R}^3 \to \mathbb{R}$ be $\mathcal{C}^1$ functions. Define

$$F(x, y) = \int_{f(x,y)}^{g(x,y)} h(x, y, t)dt.$$

Compute $\frac{\partial F}{\partial y}$.

Problem 4. Consider the equation

$$xe^y + ye^x = 0.$$

Show we can solve for x (as a smooth function) in terms of y or y (as a smooth function) in terms of x near $(0, 0)$. Compute $\frac{\partial y}{\partial x}$ near $(0, 0)$.