MAT257 Tutorial Worksheet 16

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Problem 1.

(a) Find the length of one arch of the cycloid

$$x = a(t - \sin t),$$
$$y = a(1 - \cos t).$$

(b) Find the area bounded by the x–axis and the arch.

Problem 2. Find the volume of the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$

Problem 3. Show that there is no vector field $\mathbf{F}$ on $\mathbb{R}^n$ for which the arclength of any curve γ is given by

$$\ell(\gamma) = \int_\gamma \mathbf{F} \cdot \mathbf{dx}.$$

Problem 4. Compute the integral $\int_S z dx dy dz$, where

(a) S is the region bounded by the sphere $x^2 + y^2 + z^2 = 1$ and the cone $x^2 + y^2 = z^2$.

(b) S is the region above the xy–plane bounded by the sphere $x^2 + y^2 + z^2 = 1$ and the cone $x^2 + y^2 = z^2$.

Problem 5. Let $\gamma : [0, 1] \to \mathbb{R}^n$ be a curve with $\gamma(0) = a$ and $\gamma(1) = b$. Show that

$$\ell(\gamma) \geq |b - a|,$$

where $\ell(\gamma)$ denotes the arclength of the curve γ.

Problem 6. Let V be a finite dimensional vector space, and let $\omega_1, \ldots, \omega_k : V \to \mathbb{R}$ be functionals. Show that $\omega_1, \ldots, \omega_k$ are linearly dependent if and only if $\omega_1 \wedge \ldots \wedge \omega_k = 0$.