MAT257 Tutorial Worksheet 15

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Problem 1. Compute the pullbacks of the following differential forms under the given map F.

(a) $\omega = \sqrt{x}dx$ under $F : [0,1] \to [0,1]$ given by $F(t) = t^2$.

(b) $\omega = ydx - xdy$ under $F : \mathbb{R}^2 \to \mathbb{R}^2$ given by $F(s,t) = (st, s+t)$.

(c) $\omega = x^2dx + ydy + xydz$ under $F : \mathbb{R} \to \mathbb{R}^3$ given by $F(t) = (e^t, \cos t, t^2)$.

(d) $\omega = dt$ under $F : \mathbb{R}^3 \to \mathbb{R}$ given by $F(x,y,z) = x + yz$.

Problem 2. Compute the line integrals:

(a) $\displaystyle\int_0^1 \sqrt{x}dx$

(b) $\displaystyle\int_\gamma x^2dx + ydy + xydz$ where $\gamma : [0,1] \to \mathbb{R}^3$ is the curve $\gamma(t) = (e^t, \cos t, t^2)$

Problem 3. Compute the line integral of $x^2i + 2yzj + y^2k$ from the origin to the point (1,1,1) along the following paths

(a) $(0,0,0) \to (1,0,0) \to (1,1,0) \to (1,1,1)$.

(b) The straight line from $(0,0,0)$ to $(1,1,1)$.

(c) What is the integral around the loop going out from (a) and coming back along (b).

Problem 4. Let M be a manifold and let ω be an exact 1–form on M, i.e. $\omega = df$ for some $f : M \to \mathbb{R}$. Show that if γ is a simple closed curved (i.e. not self-intersecting and a loop) , then $\displaystyle\int_\gamma \omega = 0$.

Problem 5. Let $\gamma : [0,1] \to \mathbb{R}^n$ be a curves with $\gamma(0) = a$ and $\gamma(1) = b$. Show that

$$\ell(\gamma) \geq |b - a|,$$

where $\ell(\gamma)$ denotes the arclength of the curve γ.