MAT257 Tutorial Worksheet 13

Adriano Pacifico – Change of Variables practice

January 29, 2020

Problem 1. Compute the following integrals:

(a) $\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{0}\cos(x^2+y^2)dydx$.

(b) $\int\int\int_E 16zdV$ where E is the upper half of the sphere $x^2+y^2+z^2=1$.

(c) $\int\int\int_E 4xydV$ where E is the region bounded by $z=2x^2+2y^2-7$ and $z=1$.

(d) $\int\int\int_E e^{-x^2-z^2}dV$ where E is the region between the two cylinders $x^2+z^2=4$ and $x^2+z^2=9$ with $1\leq y\leq 5$ and $z\leq 0$.

(e) $\int\int\int_E zxdV$ where E is above $x^2+y^2+z^2=4$, inside the cone (pointing upward) that makes an angle of $\pi/3$ with the negative z–axis and has $x\leq 0$.

(f) $\int_0^3\int_0^{\sqrt{9-y^2}}\int_{\sqrt{x^2+y^2}}^{\sqrt{18-x^2-y^2}}(x^2+y^2+z^2)dzdxdy$.

Problem 2. Compute $\int_0^{\infty}\int_0^{x_1}\int_0^{x_1+x_2}e^{-(x_1+x_2+x_3)}dx_3dx_2dx_1$.

Problem 3. Compute $\int_0^{\infty}\frac{\log x}{1+x^2}dx$.

Problem 4. Compute the integral

$$\int\int_D\frac{dxdy}{(x^2+y^2)^2},$$

where D is the domain bounded by the circles

$$x^2+y^2-2x=0,\qquad x^2+y^2-4x=0,$$
$$x^2+y^2-2y=0,\qquad x^2+y^2-6y=0.$$