MAT257 Tutorial Worksheet 1

Adriano Pacifico – The Pink Wednesday Tutorial

Problem 1. Determine the limit as k goes to ∞ (if it exists) of the following sequences:

(a) $\big(\cos(\pi k/2),\ \sin(\pi k/2)\big)$

(b) $\big(k!/k^k,\ (-1)^k\big)$

(c) $\big(\frac{\cos(\pi k)}{k},\ \frac{\sin(\pi k)}{k}\big)$

(d) $(e^k, 1/k, 7)$

(e) $\big(\cos(Ck),\ \sin(Ck)\big)$

Problem 2. Compute the following limits

(a) $\displaystyle\lim_{(x,y)\to(0,0)} 2019x^3 - xy^{2019}$

(b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^4 - 4y^2}{x^2 + 2y^2}$

(c) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2 + y}{\sqrt{x^2 + y^2}}$

(d) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2 + y^2}}$

(e) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2 \sin^2(y)}{x^2 + 2y^2}$

Problem 3. Show the set $\{(w,x,y,z) \in \mathbb{R}^4 \ :\ e^x y + 72wz + \cos(y\sin(wz)) \geq 2019\}$ is closed.

Problem 4. Find a sequence $\{x_n\} \subset \mathbb{R}^2$ with the property that for any $x \in \mathbb{R}^2$, there exists a subsequence $\{x_{n_k}\}$ with $x_{n_k} \to x$ as $k \to \infty$.

Problem 5. (Hard) Let $\mathcal{T}$ denote the collection of open subsets of $\mathbb{R}$. Show that $\mathcal{T}$ and $\mathbb{R}$ have the same cardinality.

Problem 6. (Hard) Let $\mu : \mathcal{P}\big([0,1]\big) \to \{0,1\}$ be a function with the following properties:

(i) $\mu(\emptyset) = 0$

(ii) $\mu\big([0,1]\big) = 1$

(iii) If $\{A_n\}_{n\geq 0}$ is a sequence of (pairwise) disjoint subsets of $[0,1]$, then $\mu\left(\bigsqcup_{n=0}^{\infty} A_n\right) = \sum_{n=0}^{\infty} \mu(A_n)$.

Show there exists $x_0 \in [0,1]$ such that

$$\mu(A) = \begin{cases} 1 \text{ if } x_0 \in A \\ 0 \text{ if } x_0 \notin A \end{cases}$$