1   (a) [**2**]   The integers x, x^2 and x^3 begin with the same digit. Does it imply that this digit is 1?

(b) [**3**]   The same question for the integers $x, x^2, x^3, \ldots, x^{2015}$.

2 [**5**]   A point X is marked on the base BC of an isosceles triangle ABC, and points P and Q are marked on the sides AB and AC so that $APXQ$ is a parallelogram. Prove that the point Y symmetrical to X with respect to line PQ lies on the circumcircle of the triangle ABC.

3   (a) [**2**]   A $2 \times n$-table (with $n > 2$) is filled with numbers so that the sums in all the columns are different. Prove that it is possible to permute the numbers in the table so that the sums in the columns would still be different and the sums in the rows would also be different.

(b) [**6**]   A 100×100-table is filled with numbers such that the sums in all the columns are different. Is it always possible to permute the numbers in the table so that the sums in the columns would still be different and the sums in the rows would also be different?

4 [**8**]   A convex N-gon with equal sides is located inside a circle. Each side is extended in both directions up to the intersection with the circle so that it contains two new segments outside the polygon. Prove that one can paint some of these new $2N$ segments in red and the rest in blue so that the sum of lengths of all the red segments would be the same as for the blue ones.

5 [**10**]   Do there exist two polynomials with integer coefficients such that each polynomial has a coefficient with an absolute value exceeding 2015 but all coefficients of their product have absolute values not exceeding 1?

6 [**10**]   An Emperor invited 2015 wizards to a festival. Each of the wizards knows who of them is good and who is evil, however the Emperor doesn't know this. A good wizard always tells the truth, while an evil wizard can tell the truth or lie at any moment. The Emperor gives each wizard a card with a single question, maybe different for different wizards, and after that listens to the answers of all wizards which are either "yes" or "no". Having listened to all the answers, the Emperor expels a single wizard through a magic door which shows if this wizard is good or evil. Then the Emperor makes new cards with questions and repeats the procedure with the remaining wizards, and so on. The Emperor may stop at any moment, and after this the Emperor may expel or not expel a wizard. Prove that the Emperor can expel all the evil wizards having expelled at most one good wizard.

7 [**10**]   It is well-known that if a quadrilateral has the circumcircle and the incircle with the same centre then it is a square. Is the similar statement true in 3 dimensions: namely, if a cuboid is inscribed into a sphere and circumscribed around a sphere and the centres of the spheres coincide, does it imply that the cuboid is a cube? (A cuboid is a polyhedron with 6 quadrilateral faces such that each vertex belongs to 3 edges.)