International Mathematics TOURNAMENT OF THE TOWNS

Junior O-Level Paper

Spring 2015

- **1** [3] Is it possible to paint six face of a cube into three colours so that each colour is present, but from any position one can see at most two colours?
- **2** [4] Points K and L are marked on side AB of triangle ABC so that KL = BC and AK = LB. Given that O is the midpoint of side AC, prove that $\angle KOL = 90^{\circ}$.
- **3** [4] Pete summed up 10 consecutive powers of two, while Basil summed up several first consecutive positive integers. Can they get the same result?
- 4 [4] A figure, given on the grid, consists of a 15-step staircase and horizontal and vertical bases (see the figure). What is the least number of squares one can split this figure into? (Splitting is allowed only along the grid).



5 [5] Among 2n+1 positive integers there is exactly one 0, while each of the numbers 1, 2, ..., n is presented exactly twice. For which n can one line up these numbers so that for any m = 1, ..., n there are exactly m numbers between two m's?